Including students who need special educational support in mainstream schools brought new challenges to teachers. Assuming an interpretative approach and an intrinsic case study design, we focus on the adaptations so that two 12th grade Deaf students could learn mathematics with their classmates. The participants were the Deaf students, their classmates, and their mathematics and special education teachers. Data collecting instruments were observation, interviews, informal conversations, students’ protocols and documents. Data were analysed through a narrative content analysis from which inductive categories emerged. Results focus on five interactive patterns used in classes. They facilitated the mathematics communication and learning of the two Deaf students and their inclusion.

INTRODUCTION

The sound cultural diversity that characterises Portuguese schools grew in the last decades (César & Oliveira, 2005). It brought additional challenges and responsibilities to teachers (César, 2009, in press). They are expected to rethink the curriculum and their practices having in mind students’ characteristics, needs and interests (Allan & Slee, 2008; César & Ainscow, 2006). Vulnerable minorities need special attention and inclusion should fit the different characteristics of the countries and cultures (Timmons & Walsh, 2010). National and international policy education documents stress the need to promote a more inclusive education (ME, 2008; UNESCO, 1994). But despite legislation, students needing special educational supports still face barriers (César & Ainscow, 2006). Deaf students experience high underachievement and school dropout rates and they are one of the smallest groups at Lisbon University (Almeida, 2009). Due to their specific communicational characteristics their hearing peers and teachers undervalue their (mathematical) performances (Borges, 2009; Borges, César, & Matos, 2012).

The designation of inclusive education is often used. It assumes different meanings. All of them concern social justice, equity in the access to school achievement and the promotion of students’ participation in school activities, namely in mathematics (César & Ainscow, 2006). But the focus and the ways to achieve these goals are different. Ainscow and César (2006) designate these differences as “a typology of five ways of thinking about inclusion” (p. 233). We assume a position connecting the last and third ways of thinking. The aim is to achieve a quality education for all (the fifth way of thinking). But to achieve this, we need to pay close attention to all vulnerable groups and to promote equity (the third one). As Clapton (2009) claims,
we need a transformatory ethic of inclusion, rupturing with the previous concepts of disability and inclusion. Thus, we consider the recommendations from UNESCO (1994). We conceive school inclusion/exclusion as a contribution to students’ life trajectories of participation which are shaped by inter- and intra-empowerment mechanisms developed – or not – through school practices (César, 2012).

Adapting the curriculum to each and every student (César & Santos, 2006; Rose, 2002), respecting and valuing their participation in different cultures (César, 2009, 2012, in press), and allowing them to give a meaning to school knowledge (Bakhtin, 1929/1981) is more striking when it comes to mathematics. This subject is associated with high academic underachievement, rejection, negative social representations and low positive self-esteem (Machado & César, 2012). Giving a meaning facilitates knowledge appropriation and the transitions between contexts, scenarios or situations (Abreu, Bishop, & Presmeg, 2002; César, 2009). It involves reflecting on classroom practices, including the nature of the tasks, working instructions, interactive patterns, didactic contract, evaluating system, regulatory dynamics and inter- and intra-empowerment mechanisms (César, 2009, 2012, in press). Thus, every teacher can use the curriculum as a vehicle for inclusion or as a lever for exclusion (Rose, 2002).

Policy education documents point to mathematics communication as one main goal (e.g., Abrantes, Serrazina, & Oliveira, 1999; NCTM, 2000). Elaborating and testing conjectures, producing sustained argumentations, establishing connections, or being critical about mathematical issues regarding society are significant aspects in mathematics learning (Alrø, Ravn, & Valero, 2010; Matos, 2009). A communicational common basis is needed, creating intersubjectivities and making mathematical messages understandable (Borges & César, 2011, 2012; Borges, César, & Matos, 2012). Students need learning opportunities, support and adaptations that are adequate to their uniqueness, including their cultural diversity, that facilitate meaningful mathematics learning and proficiency. Assuming a historical-cultural approach and knowing students’ zone of proximal development (Vygotsky, 1934/1962) facilitates the promotion of transitions from their solving strategies and ways of thinking into more formalised mathematical conceptualisations (César, 2009; César & Santos, 2006; Roth & Radford, 2011).

Like Sfard (2008), we assume learning and thinking as communicating. Thus, social interactions play an essential role in mathematics education and teachers’ practices need to promote students’ participation and their engagement in school mathematics activities. Investigating adaptations performed when Deaf students participate in mainstream classes assumes relevance as Deaf experience particular communicational barriers that often compromise their school achievement (Borges, 2009). They must communicate and think mathematically and be able to make transitions regarding their (mathematical) knowledge, abilities and competencies.
METHOD

The problem that originated this research regards the barriers to communication and to the access to the mathematical cultural tools that Deaf students experience when included in the mainstream educational system. This work is part of a broader study (Borges, 2009). In this paper we focus in two of the four research questions: (1) What adaptations are performed by this teacher in this 12th grade class that includes Deaf and hearing teenagers?; and (2) What changes are performed by the hearing students in their communication while working and interacting with these Deaf students? These questions do not focus on mathematics learning directly, but we observed mathematics classes. We studied the participation of two pre-lingual profound and severe oralist Deaf students in their 12th grade mathematics classes: Dário and Artur (false names). They were achieving cases as they had the expected age and planned to go to university. The disclosure of successful cases contributes to a more inclusive education (Allan & Słe, 2008; César, 2009; César & Santos, 2006).

We assume an interpretative approach (Denzin, 2002) and an intrinsic case study design (Stake, 1995). The participants were these two Deaf students, their classmates, their mathematics teacher (Mariana) and their special education teacher. Mariana had taught other Deaf students before and she was particularly sensitive to their needs. The data collecting instruments were participant observation (audio recorded and registered in the researcher’s diary), interviews, informal conversations, students’ protocols and documents. The observation included the attendance of one class per week (November to June, a total of 17 classes). The numbers in the codes refer to the observed lesson – 1 to 17. The contents were mainly functions. Data treatment and analysis used a narrative content analysis (Clandinin & Connelly, 1998), starting with a floating reading. More in-depth readings included the search of interactive patterns. Inductive categories emerged (César, 2009), such as the interactive patterns used in mathematics classes (Borges, 2009).

RESULTS

The analysis of some episodes and empirical evidences allowed for the recognition of five interactive patterns used in the mathematics classroom communication: (1) spatial regulation; (2) working rhythm regulation mechanisms; (3) reinforcement schemes; (4) tutorial co-construction; and (5) clarification of doubts. These patterns played an important role in the inclusion process and in mathematics knowledge appropriation.

Spatial regulation

A teacher can walk around the classroom, be in a backlit position, and speak while writing on the blackboard or consulting a book without stopping a hearing student from following his speech, such as in the examples illuminated by Machado and César (2012). But as an oralist Deaf student uses lip reading as his/her main way of communicating with hearing people, a simple rotation of the face, a misarticulated word or a too speedy sentence breaks the communication. Mariana’s ways of acting
illuminate she was concerned with these details: “Mariana mentions the number of the lesson and dictates the summary. (...) She repeats near Dário (...) slower. She does the same near Artur” (15th lesson, May 13, 2009, p. 137). These were essential features for these two Deaf students. A less rigorous diction or the omission of syllables turn lip reading into an impossible task. This was an essential move to allow these Deaf students’ access to mathematical cultural tools. Their hearing classmates also used adaptations in communication and so enable peer interactions: “Núria, who arrived a little late, asked Dário about the summary. He does not understand and she repeats only the word summary, rotating completely her face towards him and speaking the word a little slower” (15th lesson, May 13, 2009, p. 138).

In the communication between Deaf and hearing people, the oral information can be complemented with gestures and/or other visual aids. Besides the blackboard, we observed the use of technologies like the viewscreen, the interactive board, and the graphing calculator. In one class, while using a computer program that allows visualising the image of a graphing calculator in the interactive board, we registered: “Mariana starts giving instructions about the definitions of the calculator, exemplifying in the projection in the interactive board. Students repeat the procedures in their calculators” (11th lesson, April 22, 2009, p. 109). Thus, the oral instructions were complemented with the use of the virtual calculator, facilitating mathematical learning. These resources are useful for any student but they are particularly important for the Deaf, as sight is their privileged means when communicating, and as Sfard (2008) underlined, communication is a main mediating tool for mathematics learning. These complements benefit the hearing and Deaf students as intended by the inclusive education approach (Borges, 2009).

**Working rhythm regulation mechanisms**

In these mathematics classes some working rhythm regulation mechanisms emerged and played an important role in students’ engagement in these school mathematics tasks. This teacher used them often. Those mechanisms were similar both for Deaf and hearing students. But they were used much more often with the Deaf, as she knew that the communicating characteristics of the Deaf may exclude them from what is going on in the class, particularly in whole group discussions and, as stated by Borges (2009), they also get distracted more easily.

Mariana [to Artur]: Haven’t you done [exercise] b?

Artur: That’s for homework.

Mariana: For homework? Oh, you are always watching the clock! Then, write it down. Your homework is the [exercise] 300, Paragraph b, c, and d; Test 9, Page 14. [The bell rings. Mariana speaks to the whole class]


Besides the instructions given specifically to the two Deaf students and to the whole
class, the teacher chose to ask students about their progression in their solving strategies to promote their working rhythm. Thus, instead of telling them to work or be quiet, Mariana led the students’ attention to the mathematical tasks and alerted them, in a subtle way, whenever they needed to work faster.

Another mechanism to regulate the working rhythm had to do with teacher’s positioning. By moving around between the students’ desks while they were doing autonomous work, Mariana could get closer to them and see how they were progressing. This way of acting was more frequent with the two Deaf students as she wanted to keep them in a similar rhythm and to know if they were struggling with any difficulty. This particular attention to students’ performances is mentioned in other researches as an essential feature for school achievement, particularly in mathematics (César, 2009; Machado & César, 2012). Sometimes Mariana remained longer next to a particular student to be sure s/he would keep working. This often happened with Artur who would easily get distracted: “(…) Artur starts talking to his right side. Mariana walks by and says «Well?» and stays next to him following his work, preventing him from being distracted again” (6th lesson, February 11, 2009, p. 65).

It was curious to see that the hearing classmates would also regulate Deaf students’ working rhythm. Sometimes the classmate that shared Artur’s desk brought his attention back to work, illuminating the peer’s role in students’ performances, as also stated by César (2012). The intersubjectivity they developed enabled her to do so only using non-verbal language: “Artur has “his head in the clouds” and Melissa taps him on his shoulder and, without saying anything else, he understands the message and returns to work” (17th lesson, June 3, 2009, p. 158). From what we have observed, Artur did not feel embarrassed or displeased with these small remarks. Their special education teacher (SET) also mentioned this: “Artur accepts perfectly (...) the criticism, quotation marks, of him being inattentive, not very concentrated (…)” (SET, interview, p. 14). Thus, with the help of his mathematics teacher and peers, Artur’s working rhythm improved as well as his mathematical performances.

**Reinforcement schemes**

The mathematics teacher introduced simple, discrete and efficient reinforcement schemes. For instance, she would confirm the steps used in a particular solving strategy. The teacher could say: “Mariana [to Dário]: That’s it” (1st lesson, November 26, 2008, p. 17). Other times the students requested these reinforcement mechanisms:

[Artur asks if what he has done is correct. Mariana says it is]

Artur: Did I get away with it?

Classmates also used reinforcement among themselves, encouraging each other. Sometimes, after seeking together for an answer they would share the pleasure of finding it, as also stated in other researches (César, 2012; Machado & César, 2012).
In this episode Artur and his classmate, after discussing about the correct option for a multiple-choice exercise, participate in the general discussion:

Mariana: (...) therefore the answer is…?
Melia and Artur: It’s D.
Mariana: It’s D. [Melia and Artur celebrate by hitting each other’s right hand in the air – a “high five”] (3rd lesson, January 14, 2009, p. 38)

After discussing the task, these students felt confident to answer to a question asked to the whole class. Some authors claim this is a clear sign of their participation (Sfard, 2008). These Deaf students often volunteered to answer to questions during the general discussion. This illuminates how they felt included in the mathematical activities. This celebration illuminates a well-accomplished socialisation. Artur participates in a teenagers’ typical way of acting. We infer a high level of inclusion in their peer groups, desirable in an inclusive education (UNESCO, 1994).

**Tutorial co-construction**

As observers we often saw the elaboration of an answer or solving strategy including an interaction engaging two persons (teacher/student or student/student) or the whole class, in general discussions (teacher/class). This illuminates the central role played by social interactions in mathematics learning as stated by Roth and Radford (2011). Mariana’s interventions were mainly questions or suggestions – which constitute an interactive pattern that characterises her practices, and was also mentioned in other researches (Machado & César, 2012). There was a clear effort to avoid giving the answers to students. She preferred to give students time and space (César, 2009, 2012) so that they could find the answers by their own, allowing them to mobilise and develop their mental tools (Vygotsky, 1934/1962).

Mariana: What is the first thing that you have to do here? $U_n$ tends to what value?
Artur: This is really confusing.
Mariana: It may be confusing at first but then the conclusions are the same. Remember what we did a while ago. (...) It tends to…?
Artur: They become really small.
Mariana: It tends to…?
Artur: -5, no?
Mariana: No. (...) Try to find it using the calculator. [Mariana goes near Artur and help him constructing the graph in the calculator.] (4th lesson, January 21, 2009, pp. 45-46)

The teacher does not contradict Artur when he says that this content is confusing. She tells him that it can be confusing only in the beginning, which implicitly conveys the message that she believes he will understand that topic if he goes on trying. Implicit messages are very strong elements in (mathematics) learning as well as in students’ commitment/rejection towards it (César, 2009; César & Santos, 2006). Implicit
messages are essential regarding these students’ processes of inclusion.

Another detail is the improvement of Artur’s mathematical communication, an essential feature according to Sfard (2008). He states the succession tends to “really small” values. Mariana, without criticising him, repeats the question asking for an accurate answer. When Artur guesses one value, Mariana could have given him the answer. But she suggests he should try to find the expected number in his graphing calculator. She continues to push Artur to find the answer by himself and, once again, that brings an implicit message: she believes he can find the answer on his own and improve his mathematical performances. She believes that he can learn – an essential aspect to achieve students’ engagement as also stated by César (2009, 2012) and Roth and Radford (2011).

Clarification of doubts

As we mentioned before, during the moments of autonomous work the mathematics teacher used to walk around between the students’ desks. By doing so, two patterns of doubt clarification emerged, according to who initiated this interactive pattern: the teacher; or the students.

Dário raises his arm. Mariana does not notice it and goes near Artur to become aware of his progress [regarding his work]. Dário lowers his arm. Mariana clarifies another student’s doubt and when she has finished Dário raises his arm again. Mariana goes near him and confirms what he has already done and the next step as Dário asks her if his idea is correct, or not. (2nd lesson, January 7, 2009, p. 29)

We can infer a safe class culture characterised by tolerance and the absence of a competition level harmful for the students’ learning. When Mariana is unaware Dário had requested her help, she first walks and goes near another classmate. Thus, Dário lowers his arm, waits, and when she is available he calls her again. This happened without any manifestation of unpleasantness and he keeps on working while waiting.

During the observations we also realised that students shared what they knew and co-constructed their answers in a similar way as described by Roth and Radford (2011). Usually they only requested their teacher’s help when they could not go further on their own. The teacher respected these clarifications of doubts among classmates: “[Mariana] comes back, near Dário, who is talking to Melissa about the exercise. She waits until Melissa finishes her explanation and only then she participates in their discussion” (5th lesson, February 4, 2009, p. 57). With this kind of acting the teacher encourages autonomy (an essential competency for students who are preparing themselves to go to higher education or to start working), promotes mutual help and respect, creating more inclusive spaces and times.

Sometimes individual clarification of doubts could originate useful contributions:

[Dário looks a little longer to the resolution in the blackboard, while he bites a nail and says to Núria, with a worried look]

Dário: I didn’t understand! [Mariana is explaining something to Alexandra and when
she returns to the blackboard she adds the rule for deriving the exponential [function]. Dário makes a face that seems to tell us that this detail was what was missing for him to understand the solving strategy] (9th lesson, March 25, 2009, p. 90)

The doubt of Alexandra gave rise to a collective enlightenment. It led the teacher to infer that remembering the exponential function derived rule was probably going to benefit other students. Looking at Dário’s facial expression, she was right.

Final remarks

The results illuminate a well-accomplished inclusion process regarding these two Deaf students, both as mathematics students and as youngsters in peer groups. This class constitutes an example of what is recommended by UNESCO (1994). Deaf people tend to have some difficulties due to the communicative characteristics associated with profound and severe deafness. But, in these cases, they participated in their peers’ extra-classes activities, like going to esplanades or to the movies which illuminates their inclusion process and socialisation, as stated also by César and Santos (2006) in another research.

The main adaptations this mathematics teacher introduced in her practices had to do with her spatial positioning and the care with the speed and articulation of words. It was intended for Dário and Artur to participate in the mathematics classes. Knowing the access to the Portuguese oral language is a further challenge to the Deaf, she often passed by their desks, making sure they were progressing in the tasks with an adequate rhythm, and that they were not getting blocked by linguistic barriers, such as an unknown word, or a new mathematical designation. Perhaps because it was a 12th grade class, we were unaware of changes in the nature of the tasks or in their instructions. Nevertheless, there was clear concern, both from the teacher and the classmates, to make Dário and Artur feel like legitimate participants, and to respect their characteristics, interests and needs. The importance of legitimate participation in mathematics classes is also underlined by César (2009, 2012) and by Machado & César (2012).

It was interesting to observe that the hearing classmates adopted ways of acting and communicating with Dário and Artur similar to the ones used by their teacher: being careful with the articulation and speed of the oral speech, turning their face to them, simplifying the vocabulary whenever it was needed. This way of mimicking the teacher’s adaptations draws attention to the importance and influence of the role of the educator as a facilitator (or blocker) of a more inclusive education, as also mentioned by César and Santos (2006). The horizontal interactions played several roles: contributed to the development of the students’ autonomy; promoted mutual help, facilitating the inclusion of the Deaf (and other) students; helped developing aspects related to the socialization and to create a relaxed and healthy way of Dário and Artur experiencing their deafness. In short: as mentioned by Sfard (2008), communication played an essential role in mathematics learning, particularly in formal educational scenarios. We would like to stress that the inclusion of these two
Deaf students in a mainstream class was not only beneficial for them. Their presence in the classroom asked for a particular care with communication and that also facilitated hearing students’ mathematical learning. The diversification of ways of communicating made this an enriching experience for them all in what regards socialization and citizenship.

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