ARITHMETIC TEXTBOOKS AND 19TH CENTURY VALUES

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In this paper two nineteenth century Icelandic arithmetic textbooks are investigated, both written according to the late medieval tradition of libri di abbaco*, practical textbooks. The authors were influenced by the German Protestant tradition that arithmetic education was to serve ethic education, as well as by the Enlightenment. Above all, they worked for the autonomy of Iceland from Denmark. Their textbooks served the purpose of teaching young people wise allocation and yield of their resources with the overall goal of individual as well as national prosperity and financial independence.*

INTRODUCTION

Iceland was basically a medieval society at the beginning of nineteenth century. At its close it had become modernized in the sense that towns and villages had been formed, a few bridges been built and school legislation was about to be enacted. Two substantial arithmetic textbooks, written in Icelandic, were published during the century. The purpose of this paper is to explore the role of the textbooks in the development of the society, more definitely phrased:

> How did the two textbooks relate to the struggle for Iceland's autonomy from Denmark in the nineteenth century, and which values did the authors deem to serve that purpose?

The research method is historical; exploration of the historical background of the Icelandic society, by reference to papers by distinguished scholars, and examination and interpretation of the textbooks themselves. Some research has been done recently on the Icelandic heritage of arithmetic textbooks, e.g. by Bjarnadóttir (2010a; 2010b; in print), where one or both of the textbooks in concern were studied.

ARITHMETIC TEXTBOOKS

Textbooks have been written since the origin of writing. Printing facilitated their distribution greatly and printed arithmetic textbooks abound. Arithmetic textbooks fell early into a certain pattern, while authors have had some freedom on pedagogical ordering of the material; to present mathematics in context and to develop natural curiosity. Early textbooks were not written for use in institutionalized school systems and the presence of a teacher was not assumed (Howson 1995).

Libri d'abbaco

During the twelfth through the fifteenth centuries, algorithms, based on al-Khwarizmi's arithmetic, appeared in great numbers and in a diversity of languages other than Latin. There are numerous Italian *libri d'abbaco*, arithmetic textbooks written by teachers who spoke the ordinary language of the marketplace. A characteristic feature of the

abbaci is that the solution and detailed working out of the problem is always given immediately after the enunciation. The modern habit of printing only the problem and leaving the solution to the reader, or giving only the answers at the back of the book, is never found in the *abbaci*. The books contained typically an introduction to the Hindu-Arabic numeration and the accompanying arithmetic operations, in addition to mathematical techniques for business use (Swetz, 1992; Tropfke, 1980).

The protestant reform

The general approach of the Protestant Reform in the early 1500s was to suppose the population to be literate. Martin Luther's collaborator, Philipp Melanchthon, was particularly keen on nurturing mathematics education. For him, knowledge existed primarily for the service of moral and religious education and he praised mathematics for its ethical role. Most textbooks of this time were written in the vernacular with self instruction in mind while the mercantile community was the main target group (Grosse 1901).

NINETEENTH CENTURY ICELAND

By the early 19th century, Iceland was essentially a medieval society of rural habitation. The population numbered only 47,000, almost exclusively rural. Reykjavík was becoming the centre of government and a capital town. The number of inhabitants in Reykjavík was 307 in 1801. There were no schools apart from one Latin School which was a theological seminary as well until 1847.

The Icelandic Enlightenment movement, originating in Denmark where in turn it was largely derived from Protestant Germany, had made considerable efforts in publishing educational material by the 1780s. Calamities: volcanic eruptions and earthquakes, followed by bankruptcies of official agencies, caused by Danish participations in the Napoleonic wars, reduced its impact. Two substantial arithmetic textbooks were published in the 1780s, one of them by Stephensen (Stefánsson, 1785), the main Enlightenment champion. Manuscripts were also dispersed from person to person. Reading and Christian knowledge were required for confirmation from 1743 upon the responsibilities of the homes, while legislative act, adding writing and arithmetic, was first issued in 1880. Textbooks of the 1800s were therefore not aimed at children, but young people.

By the end of the century, two lower secondary schools offering general education, primary schools in the largest towns, and a handful of schools for prospective farmers and housewives and seamen, had been spontaneously established. Legislation on state support did not exist as yet except for the Latin School and one of the lower secondary schools. Pupils had to have private tuition in Danish and Latin prior to admission to the Latin School and both schools required living in student residences, demanding expenses which limited attendance. The Latin School pupils were sons of officials and farmers (Karlsson, 2000), the state lower secondary school pupils were mainly sons of

farmers (*Skýrsla um Gagnfræða Skólann á Möðruvöllum*, 1881–1905), but in its parallel private town school, a few girls were also registered (*Skýrsla um Alþýðu- og gagnfræðaskólann í Flensborg*, 1883–). The total number of pupils in each cohort in the three schools was 40–50 in 1880, or about 3% of the population (*Hagskinna*, 1997).

THE AUTHORS AND DISTRIBUTION OF THEIR TEXTBOOKS

The two nineteenth century arithmetic textbooks describe the current society and propagate ideas of how to make the most of it. Their authors both became later Members of Parliament; Jón Guðmundsson (1807–1875) during 1859–67 and Eiríkur Briem (1846–1929) in 1881–91 and 1901–1914. They did not meet at the Parliament but were both avid supporters of the leader of a movement for autonomy from Denmark, Jón Sigurðsson (1811–1879), Member of Parliament 1844–1879.

As young and promising men, Guðmundsson was a secretary of Reykjavík Town Magistrate in 1832–1836 while Sigurðsson was the Bishop's Secretary in 1830–1833, a post that Briem served in 1867–73. In the small community of Reykjavík, young men working as assistants to the highest authorities became knowledgeable about matters and men. The most important matter all the period from the 1830s was raising Iceland to autonomy. Sigurðsson, the leader, wrote papers about his visions of schools: primary schools, schools for seamen, farmers and gymnasia, by the Prussian model, free trade and financial independence of Iceland. Guðmundsson supported Sigurðsson duly in his newspaper and Briem gave a comprehensive overview of Sigurðsson's work in his obituary of 42 pages (Briem, 1880b). The main issues in the Parliament during Guðmundsson's terms, were autonomy from Denmark and related financial claims (Laxness, 1960). In spite of Sigurðsson's hard work, by 1874, when Icelanders were granted their own Constitution under which the Parliament became a legislative body with own budget, there was hardly any piece of road in the country, no bridged rivers, no transport on sea along the coast and only little transport to other countries (Bárðarson, 1930).

The technological changes that had taken place in the neighbouring countries had only reached Iceland to a limited extent by the turn of the 20th century. Education for the common people was much slower to emerge in Iceland than e.g. in Denmark, and there were relatively fewer towns than in most other European countries. This may be seen as a favourable set of conditions for the reception of 'enlightened' ideas (I. Sigurðsson, 2010). Both Guðmundsson and Briem were influenced by Stephensen's textbook, and his writings about national economy, advocating the profitability of sheep farming exceeding cattle farming and fishing (Bjarnadóttir, in print).

Jón Guðmundsson

In later life Jón Guðmundsson became best known as newspaper editor supporting J. Sigurðsson. Due to bad health and poverty, Guðmundsson had become 25 year old

when he completed the Latin School. Serving as overseer of the King's properties in South-East Iceland he wrote an arithmetic textbook. *Reikníngslist, einkum handa leikmönnum* / *Arithmetic Art mainly for laymen*, published in 1841, well before he became known for his participation in the struggle for Iceland's autonomy from Denmark. The content of the book is traditional, but its characteristics are his sincere advice to young people on avoiding squandering on luxuries like coffee, sugar and spirits. It was only printed once and used one year only in the Latin School. Other schools had not yet been established. No sources have been found on its use, but as the only available printed textbook for 28 years, it is not unlikely that some number of farmers' sons who had no choice of schools, tried to learn from it.

Eiríkur Briem

Eiríkur Briem (1846–1929) became professor of theology at the Theological Seminary, later University of Iceland, established in 1911. He was also the guardian of the first bank, The National Bank of Iceland, established in 1886, and was entrusted to a number of tasks concerning Iceland economy in the early 1900 (Bárðarson, 1931). Briem did most of his secondary schooling and the Theological Seminary education, staying at home, teaching his siblings. He was only 23 years old and had not been ordained when he published his arithmetic textbook, *Reikningsbók*, in 1869. The textbook was expanded in 1880 into two volumes. The first volume was republished ten times until 1911 and the second one four times, thus being the main reference book in arithmetic for up to half a century. Both volumes were used for teaching at both the lower secondary schools (*Skýrsla um gagnfræðaskólann á Möðruvöllum*, 1880–1905, *Skýrsla um Alþýðu- og gagnfræðaskólann í Flensborg*, 1883–) and the first one at the Latin School (*Skýrsla um hinn lærða skóla í Reykjavík*, 1875–1983). When primary schools became more common from the 1860s, it was recommended as a handbook for teachers but too difficult for children.

THE CONTENTS OF THE ARITHMETIC TEXTBOOKS

Table 1 provides a list of the typical content of the *libri d'abbaco* (Van Egmond 1980) with columns for comparison to the contents of the two arithmetic textbooks: *Reikníngslist* (Guðmundsson, 1841), and *Reikningsbók* (Briem, 1869, 1880).

	J.G.	E.B.
1. Arithmetic:		
Numeration: the four arithmetic operations, applied successively to whole numbers, fractions and the compound quantities of monies, weights and measures	X	X
Tables: multiplication tables for numbers and monetary units, tables of squares and list of the parts of monetary units;	X	X

2. Business problems:		
Finding the price or amount of goods, usually by the Rule of Three	X	X
Currency exchange, and conversion of units of measurement	X	X
Problems relevant to bartering	X	X
Partnership dividing profits between members of partnership	X	X
Interest and discount – distinguished between simple interest and compound interest, computed by percentages	X	X
Equation of payments – a series of loans combined for repayment on a single date		
Alligation – metals of varying purities combined in a mixture		X
3. Recreational problems; often including algebraic questions		X
4. Geometrical problems		X
5. Methodological section		
Rule of Three or the ‚golden rule‘	X	X
Rule of the false position, rule of the double false position		
Algebra, solution by an unknowns and equations		X
6. Miscellanious – number theory, tarifs, calendars, etc.		X

Table 1: Comparison of Guðmundsson's and Briem's textbooks to the *libri d'abbaco*

The content of Guðmundsson's *Arithmetic Art* was confined to the above list, while Briem's *Arithmetic*, in the first edition of its second part, contained more advanced material, such as logarithms for quick backwards computations of compound interests. That the sections of algebra, equations and logarithms were excluded in later editions of the book, witnesses the fact that the general public, who were the readers of the book, did not require it and may not have wanted to spend money on unnecessary information. Books had to be kept at minimal price.

Both authors mention foreign textbooks, e.g. by the Dane Georg F. Ursin, which both may have studied at the Latin School. However, the contexts of their books do in both cases concern the Icelandic farming society and its economic issues.

THE MESSAGES OF THE TEXTBOOKS

The two textbooks, published by an interval of a quarter of a century, describe similar societies, the old farming society of rural inhabitation. The farmers went together with their tenants and servants to the fishing grounds in the spring fishing season. They allocated the revenues of their farming and fishing and dealt with the merchants. Most of the trade was pure barter where no money was present although the prices had been

calculated in money since 1776 (Karlsson, 2000, p. 244). The farmer deposited his products: wool, meat, fat, fish, knitting and weaving products, and feathers. Against this he drew on his account what he needed. The society was to large extent built up as self-sufficient, and the necessary goods were often confined to grain, such as rye, oats, etc. In addition there were luxurious goods, such as sugar, coffee and spirits. The farmer was also to pay his taxes and various official payments. An important part was the support of pauper and invalids, which expenses had to be computed to a certain degree of exactness.

Each category of arithmetic: the four arithmetic operations by simple numbers, composite numbers (with monetary or measuring units) and fractions; the Rule of Tree, simple and inverse; percentages and interests; was followed by six to ten story problems by Guðmundsson. The majority of them concern the above transactions by the merchant, clearing up death estates and heritage, etc. In most categories one meets one or two problems containing advice on sensible allocations of resources.

Briem had many more problems and their solutions were generally left to the reader except the very first ones in each sections which served as examples. His problems containing advice and warnings were located in the latter part of the 1869 edition in the sections of percentages. In the 1880 edition the content had been split into two volumes and the ethical advice was exclusively contained in the second volume along with the inverse Rule of Three, percentages and interests. Several recurring themes of the two textbooks are recited below.

Avoid borrowing

Both authors warn seriously against borrowing. An example tells of a man who borrows money for two years by accepting an obligation of 100 kronas which only gives him 70 kronas. Furthermore, he has to pay 4% rent p.a. In fact he has to pay 27 1/7 % p.a. (Guðmundsson, 1841, p, 241). Usury was clearly not invented in the 21st century. The author adds a comment following this and another similar example:

> These 2 examples exhibit how unscrupulous some people are by utilizing others' hardship, and how these in trouble, often out of distress and sometimes out of ignorance, submit to high rents. (Guðmundsson, 1841, p. 243)

By Briem's opinion, borrowing was not appropriate to anything that did not provide revenue (Bárðarson, 1931):

> A farmer borrowed autumn-wool by the end of March against paying it back by the end of June by the same amount of spring-wool; now the price of the autumn-wool was 22 shillings, but of the spring-wool 36 shillings; how much did he have to pay in percentages p.a.? Answer: 254 6/11% p.a. (Briem, 1869, p. 131)

The farmers had to ensure enough wool for their family and servants to spin for knitting and weaving in the late-winter and early spring before the bustle of lamb-births, shearing and hay-making; the old farming cycle in a nutshell. Therefore, the

farmer had two options; either to keep his people idle for several months or to borrow wool at high prize. The latter was a better option as the home-made products would increase the value of the wool considerably.

Improve farming and avoid spending on luxurious goods

Briem was much occupied with teaching the youth how to make the most of traditional farming and avoid spending. The extended version of 1880 contains:

> A farmer calculated that his household spent 100 kronas on coffee a year. How much is that for twenty years if he could have had 5% revenue of the 100 kronas by allocating them to improve the land and increase his livestock? Answer: 3306.60 kronas. (Briem, 1880, p. 37)

Briem knew how sensitive his countrymen were to taxes and gathered that taxes on luxury spending were the most justifiable. In 1889 when the Icelandic Budget was unusually difficult after periods of pack-ice, stopping fishing, causing famine and emigration to America, Briem suggested in Parliament an extra tax on coffee and sugar as luxury (Bárðarson, 1931). This suggestion echoes his textbook example nine years earlier.

Guðmundsson (1841, p. 215) tells about a coffee-gulper who used to drink five cups of coffee a day. He bought coffee-beans that he thought would last for a year. After twelve weeks he realized it would only last 40 weeks. The task was to find how much he might drink a day and then how long the portion would last if three cups a day were consumed and six cups on Sundays. Briem (1869, p. 107) repeated the task for a whole farm where the master directed how much his servants may drink a day.

Guðmundsson also told a long story of two brothers. Both had been working as free workers for salary, including food and working clothes. This was a novelty. Before, all people had to belong to a farm, either as family or servants. The elder brother had earned more, but spent his salary on spirits, coffee, clothes, knick-knacks, etc. and had less left of his year-salary than his younger brother.

The consumption of coffee seems to have worried many responsible citizens in the 19th century. The import of coffee increased from practically nothing in 1820 to 44 tons in 1840 and to 218 tons in 1866 or fivefold (*Hagskinna*, 1997). The same applied to sugar. A contemporary source (NN1, 1849) expressed concerns that even people in the smallest cottages had to drink strong coffee, even twice or thrice a day. The farmer could not direct his servants to work, and his servants could not work, until they had had their coffee. Seafarers had to wait for coffee however the weather and other conditions were. In another article in the same issue (NN2, 1849), the writer understood well that many a good farmer was shocked to learn how much of the farm's resources was spent on coffee and its auxiliaries; the grinder, the kettle, the cups, the sweets and the cream, in addition to the firewood- and time-consumption. Coffee consumption was also discussed in the parliament (*Alþingistíðindi*, 1871).

It is by no means unique in history to resist coffee consumption. In the after-WWII period, German economy was ruined. People were urged to drink beer rather than the imported coffee which had high import duties into the 1970s.

Wise allocations and investments

Briem gave an example demonstrating that a tenant who owned 300 kronas was better off by improving and evening out a rented land for the amount in order to be able to feed more cows than else. The other option was to rent the amount to another person by 4% interest p.a. The result was that the tenant would make a profit of nearly 400 kronas exceeding the interest (Briem, 1880, p. 33–35). This example is not found in the 1869 edition of the textbook where compound interests were also introduced. Briem, as a parish minister during 1873–1880, did similarly himself. He had swamps drained on the farm belonging to his parish church, and set up irrigation on the land on his own cost. However, he was well off after his seven years' service as a parish minister (Bárðarson, 1931). Depositing money in a bank was still an unfamiliar concept. The first bank was only established in 1886 where Briem himself was involved.

Spirits

Both authors give a number of problems about spirits. People mixed the spirit with water, sometimes to lower the price (Briem, 1880, p. 2) but often to gain more profit. Guðmundsson (1841, p. 228–30) tells a long story of a shady dealer buying spirit in Reykjavík to sell in rural areas and making (too) much profit. Another example concerns a worker in Reykjavík who consumed half a pot of spirit each day from the age of thirty to his death at sixty. The expenditure was gigantic, which must have been the intention to demonstrate (Guðmundsson, 1841, p. 40). Briem was also concerned about alcohol consumption:

> In 1866, 33,029 pots of wine were imported to Iceland which was 2.2% more than the year before; how much was that? (Briem, 1869, p. 125).

In 1866 the number of household in Iceland was about 9,400, and official numbers for import of spirits were 511,700 litres and other alcoholic beverages 88,200 litres (*Hagskinna*, 1997). This means that 54 litres of spirits were consumed on average in each household and 9 litres of other alcoholic beverages. These numbers dropped suddenly in 1873 after an ordinance 1872 about a duty on spirits (Árnason, 1988). The duty was still raised in 1875 and 1879, reflected in dives in the import quantities. They rose again, but never to those before 1873. The 1872 ordinance was issued in the period between the Parliament services of Guðmundsson and Briem but was duly supported by Sigurðsson, also for morality reasons (*Alþingistíðindi*, 1871).

Health issues and social problems in connections with alcohol were not mentioned in the textbooks, only the waste of money. However, alcohol consumption was

considered a growing social problem when a branch of IOGT, the International Organization of Good Templars, was established in 1884 (Karlsson, 2009).

SUMMARY AND SOME CONCLUSIONS

Guðmundsson and Briem both belonged to the typical landless educated elite who had themselves to be economically thinking and thrifty. According to Enlightenment ideas they believed that education would 'enlighten' people to allocate their resources wisely. Their textbooks belong to the *abbacus* tradition of practical textbooks as is clearly seen from Table 1. Furthermore, the authors, both theologically educated, were faithful to the protestant tradition in including ethic education in their arithmetic textbooks. They warned people of spending money on luxurious goods, such as coffee, sugar and spirits, for their own financial prosperity but also for the society's. The homes were expected to consume their own products as was the society as a whole. Sources witness that Briem practiced in his own life and in the Parliament what he preached in his textbook. Less is known about Guðmundsson. His biography author was more concerned with his activities towards autonomy from Denmark. This was certainly the overall goal of all these men; Sigurðsson, the leader, Guðmundsson and Briem, and for that the population had to be educated and capable of allocating their own personal resources in a wise manner. The conclusion is therefore that the two textbooks by Guðmundsson (1841) and Briem (1869; 1880) were important channels for conveying ideas that the authors gathered would lead Iceland and Icelanders to prosperity and financial independence.

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