MAKING SENSE OF NEWTON'S MATHEMATICS

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*This paper describes a project conducted by the author, developed at the Bath Spa University in England, and which included teachers in training and their pupils, working alongside each other in order to make sense of Newton's mathematics. It drew on the original sources: "Newton's Mathematical Wastebook", albeit in its electronic format. The two main aims of the project were to engage teachers and pupils in a joint research enterprise, and improve on teachers' subject knowledge by asking them to prepare resources based on Newton's original work for their sessions with secondary pupils. The project described here was part of the educational remit of the **Newton Project** website aiming to put all Newton's work (and interpretations related to it) on-line.*

BACKGROUND TO THE PROJECT

The outreach for the *Newton Project*[i] consisted of engaging thirty one student teachers, attending the PGCE Secondary Mathematics course at the Bath Spa University, and thirty five pupils with whom the teachers worked (of varying ages, 13 to 18 year olds) from two mathematics departments in two Bath Schools.

The benefits of the work with the original sources from the history of mathematics have been explained at length in various publications in the past twenty years, most notably in Laubenbacher & Pengelley (1996). We particularly mention this publication, as it was the description of the excitement of learning that we wanted to initiate in our teacher students:

> As with any unmediated learning experience, a special excitement comes from reading a first-hand account of a new discovery. Original texts can also enrich understanding of the roles played by cultural and mathematical surroundings in the invention of new mathematics. Through an appropriate selection and ordering of sources, students can appreciate immediate and long-term advances in the clarity, elegance, and sophistication of concepts, techniques, and notation, seeing progress impeded by fettered thinking or old paradigms until a major breakthrough helps usher in a new era. No other method shows so clearly the evolution of mathematical rigor and abstraction.[ii]

As one of the issues in teacher education, identified by the author in previous studies,[iii] is the lack of subject knowledge and interest in learning more about the mathematical content, this approach seemed worthy of at least an experiment, and certainly worthy as an attempt to bring Newton's mathematics closer to school teachers and pupils.

The main aim of the project was therefore to engage both teachers and pupils to read original sources in the search for deeper understanding of mathematical concepts taught at secondary level.[iv] Whilst the mathematics which is being taught and practiced now is quite different to that which Newton knew and developed, a correlation was established between the two through attempts to read diagrams, equations and Newton's explanations of his thought processes in the development of calculus. As we shall see at the end of this paper, teachers reported that the understanding of some of the most fundamental concepts of A-level mathematics became deeper and meaningful to both pupils and teachers as a consequence of their engagement in the project, and the teachers were able to prepare and extract enriched pedagogical material as examples for their pupils.

Teachers were divided into five working groups, investigating the:

1. General history of calculus
2. Introduction to the study of mechanical curves and the tangent problems
3. Development of Binomial Theorem
4. The development of the Fundamental Theorem of calculus
5. The spread of Newtonian science in Europe.

The teachers worked on 'deciphering' Newton's original manuscripts (given in electronic format) and preparing extracts of these to present to the secondary pupils. They all looked at the possible links between their topic and the topics from the A-level syllabus.

EXPECTATIONS IN THE PROJECT AND TEACHER TRAINING

'Making sense of Newton's Mathematics' was an experimental project – it tried to establish how and if, the novice teachers and their students make more sense of mathematical concepts through the learning process by having access to original works of mathematics. The expectation was not for all teachers to make use of all the resources available to them, nor to be able to explain each and every concept mentioned in Newton's *Mathematical Wastebook*.[1] We wanted, and asked teacher students, to make a correlation between what is being taught at A level (pre-undergraduate mathematics course in UK, 16-18 year olds) and what we expect our pupils to understand, with what Newton and his contemporaries really discovered and worked on.

Teachers were therefore asked to manage their workload in such a way that they familiarised themselves with all available resources, and produced one good explanation and/or resource for one concept they wanted to understand better themselves, for teaching during their practice in schools.

Mathematics teachers sometimes expect their pupils to take things for granted as not all mathematics can be explained by what is already known to them.[v] There is also a

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[1] See the online *Wastebook* at http://cudl.lib.cam.ac.uk/view/MS-ADD-04004/.

noted disparity between what actually students understand and what they think they understand (and the same goes for teachers).[vi]

Even if the question of the subject knowledge (including the historical account of the development of a concept) was not an issue with teacher trainees, there is also the issue of the lack of time to explain the whole development of a concept, and there is not often enough time to explain why exactly something works as it does in mathematics as the lesson time is limited, and the syllabus to be covered is usually substantial.

In the last several paragraphs therefore, we have introduced many issues relating to the subject knowledge (or the lack of it) in teacher students. In order to avoid vagueness, let us summarise some of the findings from the studies described at length elsewhere[vii]:

a) Teachers generally have poor level of subject knowledge (of those mathematical topics that are prescribed by the National Curriculum in Britain) at the beginning of their Teacher Training course,

b) Teachers have very poor, if non-existent, knowledge of the historical mathematics (apart from the incidental and the anecdotal)

c) Teachers have no, or very vague, knowledge of the benefits of understanding mathematical content in the context in which the concept taught was invented

Another recent study by the author[viii] also concluded that teachers are, at this time of their professional development, exploring the ways which support their exploration and discovery of mathematical concepts in a way that offers new insights, allowing them deeper understanding of the mathematics. Teachers also seem very keen at the beginning of their teaching career to undertake intense study in this area, i.e. the history of mathematics.[ix]

Having these findings in mind, and the principle of 'reorientation' as described by Furinghetti (Furinghetti, 2007), we hoped that learning mathematics from the original sources and comparing it with mathematics from the syllabus, would allow teachers to both pay attention to the development of subject knowledge, and re-position themselves in terms of the pedagogy through greater understanding of both the concept in question, its historical development, and in trying to overcome the difficulty in trying to explain the concept from two different perspectives (that of the original, and that of the syllabus-presented concept).

HOW NEWTON'S APPLE HIT THE GROUND

Majority of the pre-university syllabus in mathematics in England is based on the study of calculus (and most of other topics appearing can also be linked to Newton's work).[x] The study of curves however is devoted, almost entirely, to the study of quadratics and cubics, albeit without any attempt to consider them in the wider scheme of things either in terms of their classification or as part of wider classification of curves.[xi] For example the study of conics is now redundant in the English pre-university mathematics curriculum, so the study of parabola is entirely based on its algebraic analysis and does not mention that this too is a conic section curve.

It was considered that some historical insight into the study of curves by Newton, his persistent attempts to 'resolve problems by motion'[xii], and his study of the curves via the readings of Descartes and van Schooten's (1615-1660), and their description of dynamic generation of curves was crucial to his later work on fluxions and subsequently his formulation of calculus. To this end, an introduction to the work of van Schooten and Descartes was deemed useful. Newton built his work on, as we already said, Descartes among others. In his study of tangents, Newton looked at finding not only the one circle which touches a given curve at a given point, but to finding the 'best fit' curve – the one which would most closely approximate the curve at the given point. His work on this concept is described in *Epistola Posterior* 1676 and can be seen on the diagram in Fig. 1. The three circles there all touch the curve at *A*, but only *c1* is the one which fits the curvature of the curve at point *A*.

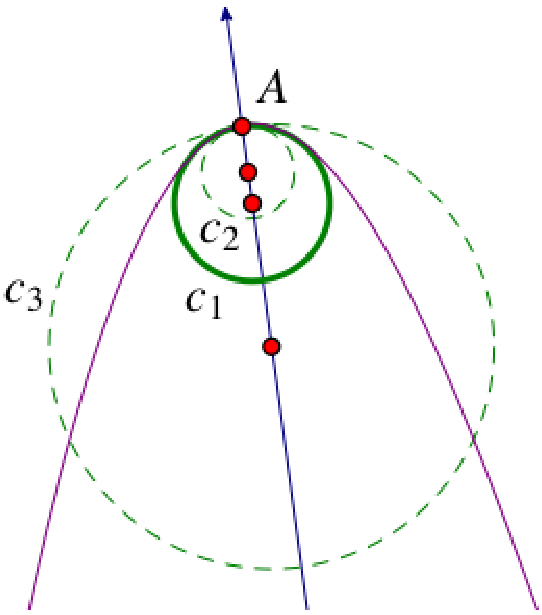

Figure 1: Newton's work on approximating the curve at the given point; see *Epistola Posterior*, 1676.

Newton uses the infinitesimal quantities to find the slope of a tangent to a curve at a given point, and calls these *fluxions*. At this point the link between the study of the curves and in particular their 'dynamic qualities' or description became apparent and the teacher students were encouraged to consider the following:

- What is the difference between geometry of Euclid and the mechanical or dynamic study of curves?
- How do we find the gradient of a straight line? How do we apply this to differentiation?
- What are Newton's fluxions?

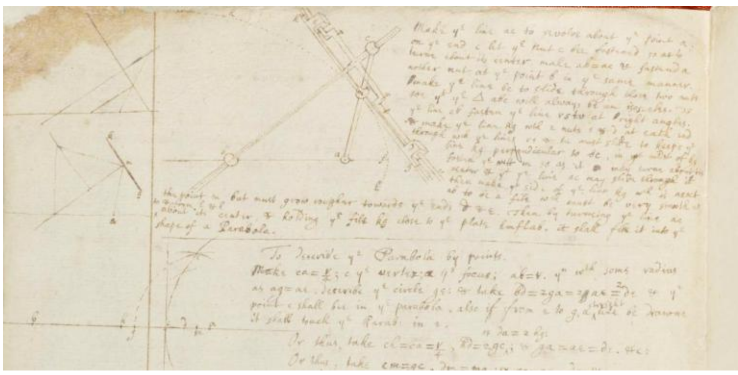

Figure 2: Newton manuscript 4004: Mathematical Wastebook - p.14. Cambridge University Library.

Further, inspired by looking at Newton's study of dynamic description of curves which he learnt of by studying van Schooten's work (as per the illustrations in Fig. 2 & 3) the student teachers created a number 'mechanical devices' both from cardboard and in the virtual world, by using dynamic geometry software.

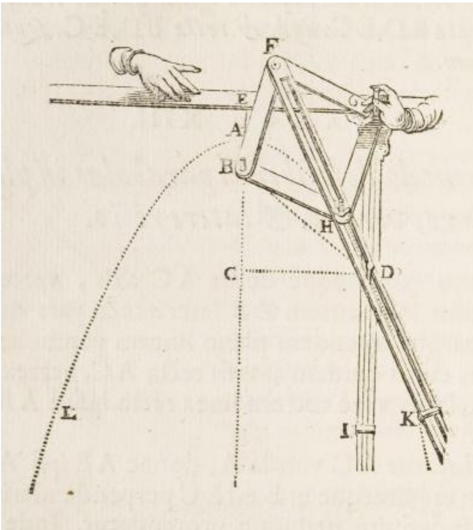

Figure 3: van Schooten's construction of the ellipse, 1646.

METHODOLOGY – GATHERING DATA

All the teachers were aware that the project would assess the suitability of the approach to both learning mathematics and teacher training. All teachers were part of the collaborative teaching cycle: they did research, lesson planning, teaching and evaluation in groups of five and six, and were then given a task to complete a reflection on the way they worked in researching the orignal sources, trying to make sense of it and structure it for the discussions and lessons with their pupils, and the teaching sessions. They evaluated how their pupils learnt mathematics by doing informal interviews with pupils and incorporated these findings into their reflections.

TEACHER STUDENTS' ASSESSMENT OF THEIR DEVELOPMENT

Whilst the teachers originally commented on the amount of time they were going to spend learning what they thought they already knew, in their reflections (anonymous) they commented on the usefulness of the exercise. Some of their conclusions are given below:

1. After the Newton Project I feel I understand the area well and can now explain it to others. The link between Newton and Binomial theorem was interesting and I did not know such a link existed before this project… I found that this helped put things into context and improve my understanding.
2. In the whole I believe this project has been a fantastic learning curve, as it consolidated skills I was strong at and strengthened those I was weak at. My research skills, for one, are an area that I lack in. My groups topic was, *The History of Calculus*, having such a topic forced me to do research allowing me to develop a skill I otherwise wouldn't have.
3. This was not only a learning curve for the pupils, but also for me.
4. The 'deconstruction' process was absolutely pivotal in creating process and breaking down the 'lesson' into manageable chunks for the students… So the 'deconstruction' process' was important for both students and teachers.

Some of the benefits the students therefore themselves identified:

- Working from original sources may lead to learning about the links between the topics teachers did not know of before, therefore enlarging their understanding of both the topics they teach, and the interrelationship between mathematical discoveries
- The research skills were improved – a necessary skill for a teacher, but one which is rarely put to the test in teacher training
- The deconstruction process on the topic is integral to the teaching, and the opportunities were all the more available when the teachers had to engage themselves with the learning process .

CONCLUSION

Whilst the evaluations of the project are purely qualitative, out of thirty one teacher students only one had a negative comment in their evaluation. This related to the fact that the student teachers are already under a lot of pressure to complete various tasks; learning about the origin of calculus this student did not deem necessary when preparing to teach it. Because the evaluations were anonymous, it is impossible to say anything about the progress of this student in the latter part of the course.

The structure of the project allowed for not only pupils in schools and teacher students to learn some new facts about mathematics, but also teacher mentors and the author of the project and the paper, learnt about some new connections between the topics, as well as the resources which they will be able to use in the future. The learning therefore occurred at all levels, promoting the model of learning in mathematics whereby the history of mathematics offers a field through which the professional learning landscape is being developed for teachers at the same time as the mathematical knowledge is being developed for their pupils.[xiii]

Out of all the outcomes, the most unexpected one was perhaps an occasion during which a student was able to help the teachers by translating the original text from Latin, thereby contributing directly to the teachers' understanding of the mathematical content. In this context, 'making sense' meant learning of teachers and students side by side, in a very literal way.

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NOTES

[i] Newton Project is an online resource developed by the team of historians of science, aiming to put all Newton's writings online, regardless of their 'discipline' classification. See http://www.newtonproject.sussex.ac.uk/prism.php?id=1.

[ii] Laubenbacher & Pengelley 1996, p. 257.

[iii] Lawrence and Ransom, 2011.

[iv] See in particular Furinghetti, 2007 and Lawrence, 2008.

[v] Nathan and Kenneth R. Koedinger, 2002.

[vi] See Knuth, 2002.

[vii] As described by Lawrence and Ransom, 2011.

[viii] See Lawrence, 2012.

[ix] As described by Lawrence, 2012.

[x] For example, and to mention only a few: number series, binomial theorem, calculus.

[xi] Whilst it is accepted that Newton's classification of quadratics and cubics is beyond the remit of this level of study, it is nevertheless the considered that the study of motion in curves – for example Newton's work on 'to resolve problems by motion' – MS. Add. 3958, fols 49-63 – were part of his view of curves, and therefore integral part of the principles of calculus.

[xii] See Ms. Add. 3958, fols. 49-63, and another version, *De Solutione Problematum per Motum*. There is evidence on this work in his *College Notebook*, MS. Add. 4000, see http://www.newtonproject.sussex.ac.uk/view/texts/diplomatic/NATP00128 (accessed 1st September 2012). See also the transcribed paper version in Hall & Hall, 2009, p. 15.

[xiii] See Lawrence, 2008.