THE DEVELOPMENT OF PLACE VALUE CONCEPTS TO SIXTH GRADE STUDENTS VIA THE STUDY OF THE CHINESE ABACUS

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The paper presents part of the findings of a study which intended to use the history of mathematics for the development of place value concepts in eighteen sixth grade Greek students. In the given pre-tests the students faced difficulties in solving place value tasks, such as regrouping quantities and multi-digit subtractions. Also they vaguely explained the carried number, a notion which is structurally associated with place value and calculations. We held an instructive intervention via a historical calculating tool, the Chinese abacus. In the post-tests students improved their scores and they often put forward expressions and symbols influenced by the abacus investigation. To a smaller extent we attempted to highlight the historical dimension of the subject. Here we present brief examples of the related activities.

Studies have shown that many students are not aware of the structure of the numbers. A great difficulty is in developing an understanding of multi digit numbers. Students need to understand not only how numbers are partitioned according to the decade structure, but also how these values interrelate (Fuson, 1990). Resnick (1983b) uses the term ‘multiple partitioning’ to describe the ability to partition numbers in non-standard ways e.g. 34 can be decomposed into 2 Tens and 14 Units. This ability is essential for competence in calculations and many types of errors have that been observed in subtraction (Fuson, 1990; Lemonides, 1994), are due to the students’ difficulty to acquire it. As a consequence, they cannot interpret the carried number, a concept structurally associated with calculations (Poisard, 2005b).

In this paper we focus on the difficulties that the students of the present study faced in the above concepts and the way that we have tried to address these difficulties with the use of the history of mathematics. Initially, we present the reasons that historical instruments may positively contribute to mathematics education. Then we describe the didactical use of the historical instrument that we used in the intervention, the Chinese abacus. Afterwards we present an overview of the intervention: the objectives, the design with the use of history and an example of a didactical session. Then a brief quantitative and a more detailed qualitative analysis of the results follow. Finally, we discuss the findings and further research issues.

THE ROLE OF THE HISTORY OF MATHEMATICS

Researchers have long thought about whether mathematical education can be improved through incorporating ideas and elements from the history of mathematics in the classroom. Tzanakis and Arcavi (2000) offer a list of arguments for integrating history in mathematics education. Jankvist (2009) distinguished these arguments between using “history-as-a-goal” (learning of the development and evolution of mathematics) and using “history-as-a-tool” (learning mathematical concepts). He also classified the manners in which history can be used into three categories; illuminations, modules and history based approaches. More precisely, modules are instructional units that vary in size and scope. They are suitable when we want to use history as a cognitive tool-since additional time is required to study more in-depth mathematical concepts-and as a goal. (Jankvist, 2009). Among the possible ways that modules can be implemented using history as a ‘tool’ as well as a ‘goal’, we find the historical instruments. As Bussi (2000) argues, they can illustrate mathematical concepts and proofs in an empirical basis. Students manufacture and explore them as historical sources for arithmetic, algebra or geometry and they may also acquire awareness of the cultural dimension of mathematics (Bussi, 2000).

Chinese abacus: A historical calculating instrument

The Chinese abacus is comprised of vertical rods with same sized beads sliding on them. The beads are separated by a horizontal bar into a set of two beads (value 5) above and a set of five beads (value one) below. The rate of the unit from right to left is in base ten. To represent a number e.g. 5.031.902 (figure 1) beads of the upper or/and the lower group are pushed towards the bar, otherwise zero is represented.

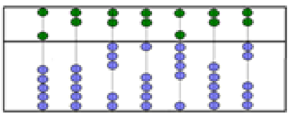

Figure 1: Representation of numbers on Chinese abacus

Many characteristics of our number system are illustrated by the abacus (Spitzer, 1942). Unlike Dienes’ blocks the semi-abstract structure of the abacus becomes apparent as the same sized beads and their position-dependent value has direct reference to digit numbers. The function of zero is represented, as a place-holder. Furthermore, it may illustrate the idea of collection, since amounts become evident in terms of place value. Finally, the notion of carried number emerges. Poisard (2005b) argues that the fact that we can write up to fifteen in each column and make exchanges with the hand, reinforces the understanding of the carried number. From the very definition of the carried number (Poisard, 2005b) its relation to the functionality of the decimal system to allow quick calculations is highlighted.

> “The carried number allows managing the change of the place value; it carries out a transfer of the numbers between the ranks”

Based on the studies about students’ difficulties and the possible positive contribution of the history of mathematics-via the Chinese abacus-in place value understanding, the present study sets the following objectives:

- To study whether sixth grade students recognize the structure of our number system when handling numbers.
- To study how they verbally explain the carried number and how they use it in written calculations.
- To study to what extent an instructive intervention with the Chinese abacus would help students handle possible difficulties and misconceptions.
- To highlight the historical context of the abacus and enrich teaching with a variety of approaches where students are actively involved.

In the present study we adopted Poisard's (2005b) proposals for calculations on the Chinese abacus. Yet, we have added new elements such as the regrouping activities. The goal was to engage students in composing and decomposing numbers in non-standard ways-as essential knowledge-(Resnick, 1983b) before implementing the subtraction algorithm. Also the decimals were not a subject matter in Poisard's research but she proposed their study through a simple adjustment of the tool. Thus, we designed activities about decimals that are not included in the present analysis.

METHODOLOGY

The study took place in an elementary school in Thessaloniki during the school year 2010-2011. The participants were eighteen twelve year old students. The criterion was that the students would be able to participate once a week during the hours when their school program was to work on a two-hour project. Four students had a very weak cognitive background while other eight often relied on procedural rules due to partial conceptual understanding.

For the first two objectives two questionnaires, as pre-tests, were administered in November. **Questionnaire A** consisted of six closed-type questions and one with verbal explanation. After the intervention similar questions were administered as post-test (Appendix II, p.10). As for the concepts regarding the creation of the questions were taken into account: a) the literature about students' difficulties b) the Greek mathematics curriculum so as to ascertain that they constitute prerequisite knowledge in the beginning of grade six, c) the feasibility of teaching via the abacus. For integers the questions concerned: name place value, expanded form, regrouping, rounding and subtraction. For decimals: pass from verbal to digit form, number pattern, addition and subtraction. Two of the questions that are subjected in the present analysis concern exchanges between classes: sub question 3b, regrouping and comparing quantities and sub question 7a, subtraction with carried number. In order to study how students perceive the concept of carried number which used in the subtraction tasks, we administered **Questionnaire B**. It consisted of Poisard's (2005b, p.101) four open questions. The same questions were given as post-test. Here we present the question: *'what is a carried number?'*.

The design of the intervention with the use of the History of Mathematics

For the other three objectives we implemented a five month instructive intervention. It was inspired by modules approach (Jankvist, 2009). We designed for this purpose a didactical sequence for the teaching of mathematical concepts. It was allocated in sections (integers, decimals and operations) and sub sections accompanied by detailed objectives and abacus activities. For every didactical session a teaching plan was elaborated including procedure, forms of work, media and material. The outcomes were recorded and several sessions were videotaped as 'feedback'. The introductory and closing activities aimed at using history mainly as a goal.

Initially, the arguments mentioned below are aimed at exploring why history would support the learning and raise the cultural dimension of mathematics. They were based on Tzanakis's & Arcavi's (2000) arguments and were grouped under Jankvist's (2009) categorization. We have entered a third category placing pedagogical arguments in an attempt to both emotionally motivate as well as develop critical thinking. Thus, students are expected to:

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A. History as tool

1. develop understanding by exploring mathematical concepts empirically
2. recognise the validity of non-formal approaches of the past

B. History as a goal

1. become aware that different people or in different periods developed various forms of representations
2. perceive that mathematics were influenced by social and cultural factors

C. Pedagogical arguments: motivate emotionally-develop critical thinking and/or metacognitive abilities

Some examples of the interrelation between the activities chosen and the arguments for such a choice are presented below. (The arguments are in parentheses). Introductory and closing activities: Presentations about number systems of the antiquity: Roman, Babylonian, Greek, Maya (B1, C); students create numbers and discuss the effectiveness of the systems (A2, B1 and C). Presentation about the ancestor of the abacus, the counting rods (B1); they form rod numerals and compare with the modern representation (A1, A2, C). Information about the abacus (B2); they compare the two forms (abacus and rods): advantages/disadvantages of practical nature, similarities/differences (B1). After the intervention they organise and present their work to the audience in the role of the teacher (C); they gather, discuss and elaborate on information about the cultural context of the abacus which led to prevail over the counting rods (B2, C) for a multicultural event. Main part: Students investigate place value concepts with abaci, web applications (A1, A2, C; Appendix III, p.10) and worksheets designed by the researchers (A2, C; Appendix I, p.10); they analyse the abacus's representations/procedures and correspond with the formal one (A1, A2); contests between pairs or groups (A1, C).

The implementation of the intervention through an example of a session

The following example is based on Poisard's (2005b) approach for subtracting on abacus with carried number. The method mainly taught to Greek schools and other European educational systems is the 'parallel additions' which uses the relation a-b= (a+x)-(b+x). The other method, the 'internal transfers', is taught in second grade as an introductory method, so it has been rarely used over the years. It allows exchanges between classes and is the only that can be implemented on abacus when using all beads. According to Poisard this method has the advantage of illustrating the properties of our number system when they have not been adequately understood.

<u>Previous knowledge on abacus</u>: regroup quantities; perform additions with carried number and subtractions without it. <u>Procedure:</u> The teacher forms the minuend of the subtraction 933-51on the web application. The number 1 can be subtracted immediately by removing one unit bead (figure 2 step1) but in the tens column the regrouping process must be put forward. A student removes one-bead from the hundreds and replaces it with two five-beads in the tens' [step 2]. Having a total 13 on the tens he removes one five-bead and gets the result [step 3].

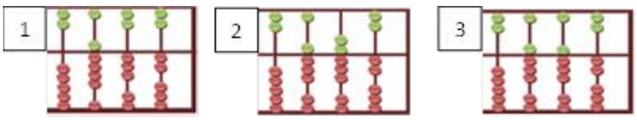

Figure 2: Example of the subtraction method 'internal transfers' on abacus

The student is encouraged to explain in terms of place value: *"I decompose 1 hundred to 10 tens and then subtract 5 tens"*. Thereafter, students work in pairs with their abacuses to solve similar subtractions. Afterwards one presents the solution on the electronic abacus while the other implements the written form explaining the process aloud.

<u>Observation from the teaching</u>: A student solved 4,005-8 initially on the blackboard. She transferred a 1 thousands' unit directly to the units' place-as if 1 thousand=10 units and found 3,007. When prompted to use the abacus, she did not make the same error. A possible explanation-positive for the process on abacus-concerns the structure of the tool; the columns may act as a visual deterrent to supplant and the hand is almost guided to make exchanges in the next column and not in a distant one. The step by step correspondence of the two processes helped her surpass the misuse.

DATA ANALYSIS AND RESULTS

Questionnaire A: The total score of questionnaire A was 100. The T tests showed a statistically significant difference between the two measurements of students' scores (t= 5.243, df = 17, p <0.001) Mean: pre-test 50.5, post-test 78.2. Particularly for the questions 3 and 7 the T tests showed a statistically significant difference between the means of the two measurements: Question 3 (t=6.172, df=17, p<0.001) pre-test: Mean 4.67, Std. Deviation 5; post-test: Mean 12.9, std. deviation 3.5. Question 7 (t= 2.807, df = 17, p < 0.05) pre-test: Mean 17.1, Std. Deviation 11.2; post-test: Mean 22.6, Std. deviation 9.0. The qualitative analysis that follows concerns, as example the sub questions 3b and 7a It aims to find if the improvement of the scores is connected with better understanding through the investigation of the abacus. For the sub question 3b were studied students' explanations.

Sub question 3b: Pre-test: 'Compare *8 hundreds 2 tens 1unit* ___*7 hundreds 11 tens 16 units* using the sign of inequality/equality. Explain the way you have thought'. **Post-test**: 'Compare *6 hundreds 3 tens 3 units* __ *6 hundreds 14 tens 13 units* using the sign of inequality/equality. Explain the way you have thought'.

Types of reasoning	pre test	post test
correct	4	16
incorrect	8	
insufficient/no explain	6	2

Table 1: Sub question 3b- Answers' reasoning analysis

Four students at the pre-test (table 1) gave correct justifications while at the post-test the majority of them were correct. Examples of students' verbal explanations in pre and post-tests follow. *Abbreviations: H=Hundreds, T=Tens, U=Units*

Correct reasoning: *'11T=1H and 1T. Also 16U=1T and 6U. So we have 700+110+16=826. The first is 826 too, so they are equal'.* Incorrect reasoning: They saw individual numbers on both sides: *'The second is bigger than the first in two numbers'.* They isolated digits and arbitrarily formed a number: *'826 less than 71,116'.* They compared the hundred's class maybe recalling a vague knowledge of upper classes: *'The first number has 1H more so it is bigger because hundreds matter'.* Insufficient reasoning: *'Because 7hundreds 11tens 16units is bigger'.*

Post-test: A figurative explanation appears (fig.5). By circling and using arrows, students were depicting the abacus process of composing ten units to a higher class.

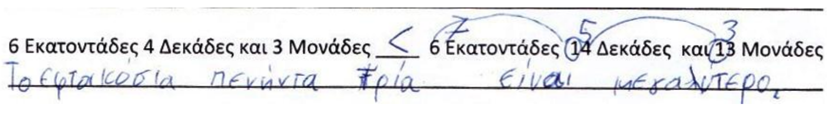

Figure 3: Sub question 3b-Example of regrouping at the post test

Translation: *'Seven hundred and fifty three is bigger'*

Other answer: *"I get 10 from 14 T and make 1 H. The H now are 7. Then we have 13 U. I take 10 U and do another 1 T. The number is 753 greater than 643"*.

Sub question 7a-Pre-test: Solve the subtraction 70,005-9 in vertical form. **Post-test**: Solve the subtraction 40,006-9 in vertical form.

	not noted carried number	**parallel additions**	**Totals**
Answers	10	6	**16**
Success	4	5	**9**

Table 2: Sub question 7a-Management of the carried number at the pre-test

Two students did not answer. From table 2 we observe that half students succeeded. The only visible method was 'parallel additions', since the rest of the students did not note the carried number. The types of errors are categorised in table 3.

Question:70,005-9	**no use of carried number**		**use of carried number**	
	N	Examples	N	Examples
Trading errors	**5**	60,008 70,010 81,098,		
Copying numbers	**1**	7,005-7		
Number facts			**1**	69,997

Table 3: Sub question 7a-Types of errors at the pre-test

The main type of errors seemed to be the management of the carried number. For example, in the answer '60,008', though the carried number is not noted, the error is the transfer of 1 thousand directly to the units' position (Lemonides, 1994).

Sub question 7a-Post-test: Almost all students succeeded and the number of students who did not use the carried number decreased because of the use of the new method that requires the notation of the carried number (table 4).

	not noted carried number	parallel additions	internal transfers	**Totals**
Answers	4	7	7	**18**
Success	3	6	7	**16**

Table 4: Sub question 7a-Management of the carried number at the post-test

The method 'internal transfers' appears (figure 4) and along with 'parallel additions' was applied successfully. Note: the method 'parallel additions' was applied mainly by students who had successfully applied it at the pre-test while the method 'internal transfers' by those who had not be able to handle the carried number correctly.

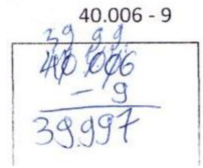

Figure 4:-The method 'internal transfers' as implemented by a student at the post test

Questionnaire B- 'What is a carried number?'

Explanations	N
find/use/ something left over in calculations	9
Example with addition	5
I don't know/remember, I cannot describe it	4
When the number exceeds 10	1

Table 5: Pre-test- The interpretation of the carried number

Explanations with the use of an example	N	Verbal explanations	N
Composing e.g.10hundreds=1 thousand	**6**	Ten units of a position move to the next position as one unit	**3**
Decomposing e.g.1hundred=10 tens	**1**	Number kept/used in operations for transfer	**2**
Composing/decomposing	**1**	Borrowing from a number	**1**
		A format of tens, hundreds etc. for transfer	**2**
		Convert a number of ten and over to another format	**1**

Table 6: Post-test- The interpretation of the carried number

The explanations with use of an example differ between the two tests (table 5& 6). At the pre-test students just performed an addition while in the post-test they put forward composing and decomposing examples. Verbal explanations at the pre-test seemed meaningless. Only in one answer we detected an attempt of mathematical explanation; *'When the number exceeds 10'*. At the post-test we can still observe a difficulty to explain but most students use the idea of exchanging; 'transfer', 'convert the format'. One is specific: *'10 units move to the next class as 1 unit'* and some mix the knowledge before and after the intervention *'number kept for transfer'*.

CONCLUSIONS

The results of the pre-tests showed that most students did not have a profound understanding of the numbers' structure; beside other difficulties that are not in the scope of this paper, they could not recognize the numbers behind a non-standard partitioning (Fuson, 1990; Resnick, 1983). Also half failed to solve a four-digit subtraction across zeros, since the structure of the numbers was not understood

(Fuson, 1990). Thus, they could not interpret the notion of carried number (Poisard, 2005b) considering it as an aid in operations but more of a vague nature. At the post test, almost all displayed a better conceptual understanding. Using schematic representations and place value explanations influenced by the activities on abacus, they successfully regrouped non-standard representations to standard numbers. As for the subtraction task, students had unsuccessfully managed the carried number in the pre-test, implemented successfully the abacus's method 'internal transfers' which requires the reverse process of decomposing numbers in non-standard way. The composing and decomposing activities on abacus and their connection to the algorithms of addition and subtraction changed their perspective about the concept of the carried number. They explained it as an exchange between classes, either verbally denoted or through a composing or decomposing example. To an extent these interpretations approached the generality of Poisard's definition.

DISCUSSION

Despite the limitations of the study, such as the small sample and the lack of experiential studies about the Chinese abacus, apart from Poisard's, the authors believe that the reasons for using the history of mathematics have been accomplished in a quite satisfactory way. By elaborating on place value concepts via the abacus, students developed understanding about the place value system on an empirical basis (literally with their hands). By analysing and discussing representations and processes with the counting rods and the abacus, they appreciated that mathematics of the past also lead to results that have logical completeness. In general, Bussi's (2000) argument was verified that in the tactile experience offered by the mechanical instruments one may find the foundations of mathematical activity.

Students were also emotionally motivated though they were going to have another maths lesson in the last hours. Especially welcomed were the activities that brought history at the forefront, although they constituted smaller part of the intervention.

As further research we suggest the study of the Chinese abacus in younger age-groups for the teaching of simpler concepts (Zhou & Peverly, 2005). Also the results about the decimals in the present study are promising and we believe that a further exploration of the Chinese abacus on this subject would be worthwhile.

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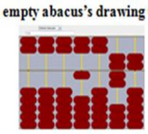
APPENDICES

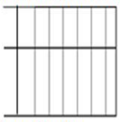
APPENDIX I

Worksheet 19

Regrouping quantities

- **Discover the number that the quantities represent on the abacus. Use the empty abacus's drawing to form the number in standard form.**





- **Explain the way you have thought**

Electronic abacus quantity 1 thousands 0 hundreds 12 tens and 10units

Standard form: =>____________

1TH 0H 12T 10U =>

Free Activity

As pairs, choose a four digit number. Use your abacuses to form it as a quantity that exceeds 9 units in some columns. Afterwards call the other teams to discover the number.

APPENDIX II

Questionnaire A-Post-Test

1 Write the place value of the digit “2” of the number 27,257,275

____________ ____________ ____________

2. Write the number 4,018,379 in the expanded form as in the colored example

(4x__________)+(0x__________)+(1x__________)+(8x__________)+
(3x__________)+(7x___10___)+(9x__________)

3. Compare the numbers using the signs >, <, =. Explain written the way you have thought

A. 4 Tens and 3 Units ____ 3 Tens and 13 Units

B. 6 Hundreds 4 Tens and 3 Units ____ 6 Hundreds 14 Tens and 13 Units

4. Round the number 5,283

a.	To the nearest ten	
b.	To the nearest hundred	
c.	To the nearest thousand	

5. Write the following decimal numbers with digits

a.	Eight tenths	
b.	Forty six hundredths	
c.	Seven thousandths	
d.	Twelve tenths	

6. Fill in the gaps using the sequence of numbers

0.2	0.5	0.8		

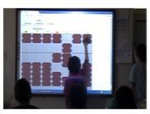
7. Perform the operations in vertical form:

40,006 - 9 5.4 + 3.46 21- 3.56 623 x 82

APPENDIX III

Abacus for demonstration

Student's abacus



Web application