# **CUBES OF CUBES**

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We present the three dimensional version of a play that is the topic of a paper sent for submission to Working Group 13. Its aim is to complete the presentation made in WG13, showing the potentiality of the play when we pass from 2D to 3D dimensions. In Cerme 8 session poster, the poster will be presented accompanying it with the same equipment utilised during the experimentation. The study of pupils' behaviours in front of the task furnishes some examples that prove the possibility of an early mathematical activity of generalization.

Keywords: kindergarten, play, generalization.

### THEORETICAL FRAMEWORK

Generalization is often cited as typical form of mathematical thinking, but without using a definition or specify its meaning.

In her analysis of the act of understanding, Sierpinska (1994) considers four basic mental operations: identification, discrimination, generalization and synthesis. Her definition of generalization is the following:

"Generalization is understood here as that operation of the mind in which a given situation (which is the object of understanding) is thought as a particular case of another situation. The term 'situation' is used here in a broad sense, from a class of objects (material or mental) to a class of events (phenomena) to problems, theorems or statements and theories" (p. 58).

Hejny (2004) write also:

"The generalisation of isolated models (experiences and pieces of knowledge) is determined by finding connections between some of isolated models. This web is the most important product of the stage of the isolated models" (p.5).

The authors present and study an example of generalization that appears during a play. It is well known that the play can promote logical and mathematical competences. Schuler (2011) highlights that:

"[...] *play* and *relationship of playing and learning* have to be explored more closely when talking about mathematics for the early years" (p.1912).

In particular, in the play utilised in this research, an important role is done to rowcolumn arrangements. Rożek & Urbanska (1998) studied in depth this topic:

"The children have a different awareness of the rows and columns arrangement. Some of them prefer rows, some of them columns. It appears that it was difficult to see both rows and columns, especially for young children" (p. 304).

## THE 'CUBE OF CUBES' CONSTRUCTION AND ITS PROPERTIES

The play named "Cube of cubes", proposed to pupils of kindergarten (5-6 years old), is based on the construction of a 'palace' of three floors (a cube 3x3x3), following these rules: "In each wall face it needs to have three different colours in each row and in each column". Pupils in groups following the indications suggested from the teacher. The materials used are: 27 wooden coloured cubes (9 red, 9 yellow, 9 blue) (Fig. 1a), their wooden support, named from children "palace" or "house with a lot of floors", and a wooden rotating disk (Fig. 1b) to facilitate gestures and the observation. In a second time, teacher removed the support and she putted the cubes one near to the other (Fig. 1c).

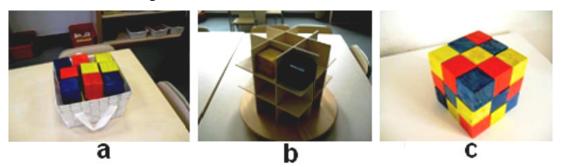


Figure 1: Materials and an example of 'Cube of cubes'.

Observing the final cube, children noticed a lot of properties, in particular they found a theorem: "In the cube there is an "internal monochromatic diagonal" and the other diagonals of cube are of three different colours". It is an example of generalization from 2D to 3D (the 2D version of the theorem is in Vighi paper, WG13, Cerme 8).



### Figure 2: The monochromatic diagonal

### References

Hejny, M. (2004). Understanding and structure, *Proceedings CERME3*, Thematic Group 3, 1-9.

Rożek, B., Urbanska, E. (1998). Children's understanding of the row-column arrangement of figures, Proceedings CIEAEM 50, Neuchâtel, 303-307.

Schuler, S. (2011). Playing and learning in early mathematics education – Modelling a complex relationship. *Proceedings CERME* 7, 1912-1922.

Sierpinska, A. (1994). Understanding in mathematics. London: The Falmer Press.

Vighi, P. (2012). The appearance of early generalization in a play. *Generalization in mathematics at all educational levels*, *Proceedings CME* 2012, Wydawnictwo Uniwersitetu Rzeszowskiego, 02/07/12-04/07/12, Rzeszow (Poland), 123-132.