SELECTING SHAPES – HOW CHILDREN IDENTIFY FAMILIAR SHAPES IN TWO DIFFERENT EDUCATIONAL SETTINGS

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To investigate the geometric competencies of children from 4 to 6 years old in two different educational settings – England and Germany – 80 children were given geometric tasks via clinical interviews. The children were interviewed at the beginning and at the end of one school year. In this paper, the results of one task – the selecting shapes task – are illustrated with the focus on children's conceptualisation of geometric shapes as well as their reasoning why certain figures were chosen as representatives of a geometric shape and why others were not.

Keywords: concept formation, geometric shapes, preschool, clinical interviews

INTRODUCTION

In the last couple of years, the importance of early learning has been widely discussed. One of the remaining questions is how the education in the early years should look like. The study at hand investigates the geometric competencies, in particular the concept formation, of children from two countries with two different concepts of elementary education: Germany (Baden-Württemberg), where learning through play and with this a constructivist view of learning is at present the main concept for kindergarten education (Schäfer, 2011; Rigall & Sharpe, 2008) and England, where the elementary education is rather systematic, curriculum based and rather instructive, and where the competencies of the children are tested via "stepping stones" which they should have acquired. There, the children enter school in the year when they have their fifth birthday, but many children go to a reception class before that. So the entering school age is about two years earlier than for children in Germany. The focus of this research is on describing the competencies the children of each educational setting acquire. As a consequence of the study, hypotheses will be formulated to what kind of competencies each way of education might lead. In this paper, one task is chosen to support some findings leading to such hypotheses.

THEORETICAL BACKGROUND

For the shape selection tasks, the children have to choose shapes belonging to a certain category (e.g. triangles) amongst other shapes. For this, the competencies that are needed are (1) being aware how a certain shape in several varieties looks like, (2) being familiar with the properties of the single shapes, (3) being able to verbally express these properties, and (4) being able to distinguish examples of shapes from non-examples of shapes. In the following, it first will be illustrated what constitutes a concept in general, followed by two theoretical models of concept development, before some empirical results of previous shape selection studies are presented.

According to Vollrath (1984) a comprehensive conception of geometric shapes is shown through being able to (1) name the shapes, (2) give a definition of the shapes, (3) show further examples of this category and (4) name all properties. However, this description was given for secondary school children. Concerning the development of such concepts, there are different suggestions as for example the two theoretical approaches proposed by Szagun (2008): First, the "semantic feature hypothesis", where the general features are learned before specific features and where the features are either present or not and apply for every member of the class, e.g. "all kinds of dogs belonging to the category "dog" are four-legged and bark". And second, the "prototype theory", which is the generally more accepted theory, some members of a category are categorised as more typical than others (Szagun, 2008). However, in order to give a complete picture of the geometric concept formation, how a concept develops has to be complemented by research findings on geometric concepts.

With the observations of Piaget & Inhelder (1975), research focusing on children's concepts of space and geometric shapes began. His topological primary thesis stated that children first realize topological features, such as "open", "closed", "interior" and "exterior". According to Piaget, the children are not able to name and to distinguish between geometric shapes before the age of six.

Another body of research has focused on children's reasoning about geometric concepts that they have formed (van Hiele & van Hiele, 1986). The van Hieles, who also created a hierarchical developmental description, constitute that children realise shapes as whole entities from the age of four onwards and are not able to distinguish shapes by their properties before primary school (from 6/7 up to 9/10). Several studies (e.g. Battista, 2007; Burger & Shaughnessy, 1986; Clements & Battista, 1992; Lehrer et al., 1998) concluded that such a hierarchic developmental description is not discrete or independent and that students also preferred different levels for different tasks. Some research also proposes that the characteristics of the single levels develop at the same time but in diverse intensity (Clements & Battista, 1992; Lehrer, 1998).

The shape selection task that will be shown in the following was already conducted in several studies (Burger & Shaughnessy, 1986; Clements et al., 1999; Razel & Eylon, 1991). To summarise the main results of these studies, most children identified circles accurately, only a few of the younger children chose an ellipse and another curved shape as circles (Sarama & Clements, 2009). The squares were also identified fairly well in these studies, between 80% and 90% of the children identified them correctly. Clements et al. (1999) found that children had some difficulties in selecting squares, for they were less accurate in classifying squares without horizontal sides (Clements, 2004). There are no circles deviating from the prototype and square prototypes only occur concerning position. Consequently, the children had more difficulties in recognising triangles which were identified correctly by about 60% of the children. Some studies (e.g. Burger & Shaughnessy, 1986 or Clements et al., 1999) revealed that children's prototype of a triangle seemed to be an isosceles triangle. The majority of children did not identify a long and narrow, scalene triangle as a triangle, although they often admitted that it has three lines and three corners.

Another research also concerning the shape selections of children was conducted by Tsamir, Tirosh and Levenson (2008) in order to examine whether there are prototypical non-examples, when children are asked to determine whether a figure is a triangle or not. They found that some figures were intuitively identified as non-examples for triangles and that more children correctly identified non-triangles as such. Another study (Levenson et al., 2011) dealt with the question what it means for preschool children to know that a shape is a triangle. They investigated whether all examples and non-examples are created equal. They found that over 90% of the reasons given by the children were based on the essential attributes of a triangle.

The study at hand complements these previous studies by investigating the competencies of children in two different educational settings and by illustrating children's understanding of geometric shapes in the light of these different settings.

EMPIRICAL STUDY

Research Questions

The underlying research questions are:

(1) Concerning the choice:
What kind of representatives do children select for a certain shape?
How do they explain their selection and how do they justify the attributes that make a figure a representative of a certain shape?

(2) Concerning the two points of investigation:
What differences can be described after a year?

(3) Concerning the two different educational settings:
What differences between the results of the children from different educational settings can be observed?

Subjects

The research gathered 81 four to six year old children, of which 34 are of English nationality and were attending a local primary school, near Winchester. The age of the children at this primary school ranges from four to eleven years. The other 47 children were from Germany and attending a kindergarten in Karlsruhe, where children from the age of three to six, up to primary school, can go.

Method

The study was conducted in the form of qualitative interviews, taking about 30 minutes each. The order of the tasks as well as the material was predetermined but in accordance with the nature of qualitative interviews this order could be altered or complemented if some of the child's answers happened to be interesting or leading into another direction worth being examined. There were altogether two points of investigation, without a special intervention, one at the beginning of the school year in October 2008 and one at the end of the school year in July 2009. Thus, it must be taken into consideration that the English children, in contrast to the German children, were instructed in geometry during the year.

Tasks

In order to investigate children's knowledge of shapes and to illustrate the concept formation of the children, different tasks were conducted in the interview of which the shape selection task – identifying and discerning shapes – will be explicitly presented in this paper. Here, the children were asked to "put a mark on each of the shapes that is a circle" on a page of separate geometric figures. After several shapes were marked, the interviewer asked questions such as: "Why did you choose this one?", "How did you know that one was a circle?", "Why did you not choose that one?". A similar procedure was conducted for squares and triangles.

RESULTS

In the following, the main results are presented by illustrating both measuring times, distinct by country. The answers of the children were thoroughly examined and then categories were generated to which the single answers of the children could be grouped to. Sometimes, the answers of the children could be grouped to more than one category, therefore the added percentages could be more than 100%. In other cases, it is not illustrated if the children didn't mark anything, so the added percentages might be less than 100%.

Identifying circles

The children were shown a picture with nine correct circles and six other shapes, like an ellipse, two twisting shapes, a semi-circle, a triangle and a square (cf. fig. 1). At both points of investigation, the majority of the children (81% of the German and 82% of the English children at the beginning and 84% of the German and 76% of the English children at the end of the school year) could distinguish between circles and non-circles correctly. In this case, "to distinguish correctly" basically means that they marked only circles as such and that they didn't select non-circles as circles. Children who could not distinguish between circles and non-circles correctly, either marked additionally other shapes or left some of the correct ones out. At the beginning of the school year, the English children either marked the ellipse (9%) and the semi-circle (3%) additionally as circles whereas the German children, although not many, chose one of the two twisting shapes (no. 8 (2%) or no. 10 (2%)).

At the end of the school year, more German (14%) than English children (6%) additionally chose the ellipse as circle. As before, the semi-circle was only chosen by English children as circle but by none of the German children.

The reasoning of the children could be grouped into the following categories (cf. tab. 1): (1) no justification, (2) visual (referring only to the appearance of a circle), e.g. "it looks like a circle" or "has (hasn't) the shape of a circle" or (3) expressed by gestures "is not a circle, for it looks like this" (then they were for example drawing the shape with their fingers into the air)", (4) using comparisons to other shapes or objects, e.g. "is not a circle because it looks like an egg", (5) naming properties, e.g. "this is a circle, because it is (isn't) round" or "is not a circle for it has corners" or (6) using the proper terms for the shapes, where the children just named the shapes: "this is a circle" or "this is a semi-circle and that is a triangle". Whatever justification was chosen, it was always correct, meaning that if a child used for example gestures to justify a circle it was drawing a circle with the fingers but not drawing another shape.

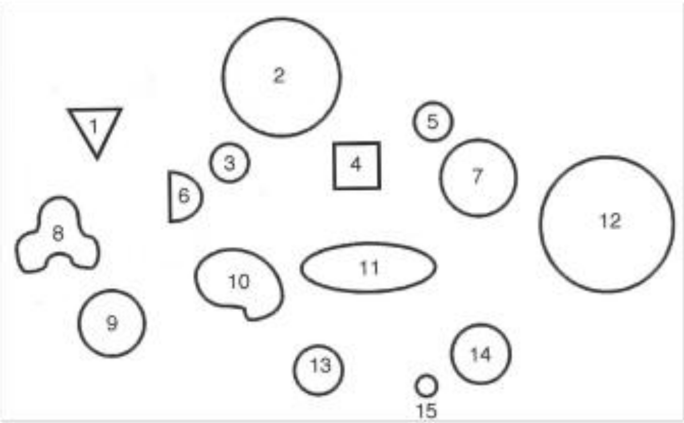

fig. 1: Identifying circles (Razel & Eylon, 1991, in Sarama & Clements, 2009).

	(1)		(2)		(3)		(4)		(5)		(6)	
	G	E	G	E	G	E	G	E	G	E	G	E
2008	9%	12%	0%	9%	19%	18%	23%	15%	21%	26%	5%	21%
2009	0%	3%	5%	6%	2%	9%	47%	15%	60%	18%	12%	38%

tab. 1: Justifications for circles

Altogether, the German children used more comparisons than the English, especially at the end of the school year. At the first point of investigation, about the same amount of children from both countries used gestures and slightly more English children were mentioning properties to explain their choices. At the second point of investigation, more English than German children used gestures and far more German than English children used properties in order to justify their choices.

Identifying squares

At this task, the children were asked to put a counter on all the squares they see. Nearly half of the children in both countries (47% German children and 44% English children) marked only all correct squares at the first investigation. At the second point of investigation, more than twice as many German (44%) than English children (21%) marked all correct squares and nothing else (cf. tab. 2).

At both investigation, clearly more English than German children (more than twice as many at the end of the school year) marked only squares but not all of them. They mainly marked either only horizontal lying squares leaving out squares in other

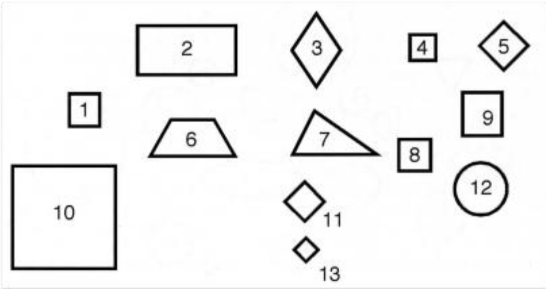

fig. 2: Identifying squares (Razel & Eylon, 1991, in Sarama & Clements, 2009).

positions (no. 5, 11 and 13) or marked only all the horizontal lying square plus one square in another position (no.5). They justified this choice by either saying that "this is too aslope to be a square" or "if you turn a square it becomes a diamond" and that "only in this (horizontal) position it is a square". The choice of the additional in another position lying square no. 5 was justified by for example highlighting that it (no. 5) "is more alike" the horizontal lying squares and "looks different than the other two shapes (no. 11 and 13)".

	Only all the squares		Only squares but not all of them		All the squares and other shapes		Not all squares and other shapes	
	G	E	G	E	G	E	G	E
2008	47%	44%	28%	53%	16%	3%	9%	0%
2009	44%	21%	28%	68%	16%	9%	9%	3%

tab. 2: Selecting squares

Clearly more German than English children at both investigations marked all the squares and other shapes. The other additional shapes that were marked by the German children were mainly rectangles (16% at the first investigation and 21% at the second investigation) but also the diamond (no.3), which was selected by 14% at the beginning and by 12% at the end of the school year. The trapezoid was selected by only one German child at the first and by two children at the second investigation but by none of the English children. In order to briefly summarise the justification of the children, it can be stated that they either reasoned by mentioning (1) the visual appearance or through (2) gestures, (3) comparisons to other shapes or objects, (4) describing their properties (informally or formally) or (5) just naming them with the geometric term. One result is for example that far more German (16% at both investigations) than English children (3% at both investigations) used comparisons in order to justify their choice of examples and non-examples of squares.

Identifying triangles

Here, the children were asked to put a counter on all the triangles they see in the picture (cf. fig. 3). Altogether, there were six triangles (no. 1, 4, 6, 8, 11, 13), two shapes consisting of triangles (no. 10, 12) and six non-triangles (no. 2, 3, 5, 7, 9, 14). The triangle selection task was more demanding than the circle or square selection task, because there were less "intuitive non-examples" (e.g. Tsamir et al., 2008), meaning shapes that are clearly no triangles, but more "non-intuitive non-examples" or "nearly triangles", lacking one attribute as for example straight sides (no. 3, 5, 7 and 14) or three corners (no. 9). Thus, the children had more difficulties when asked

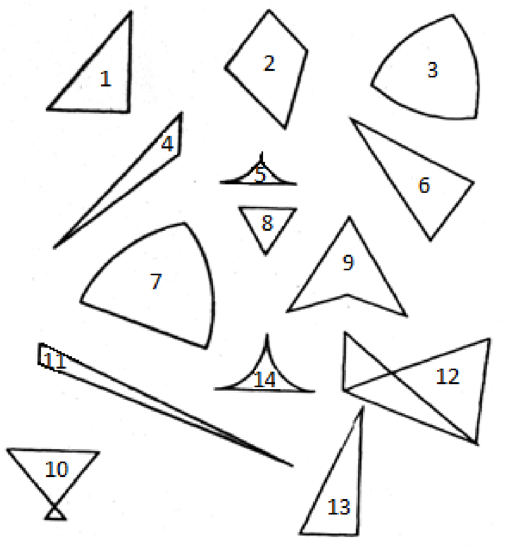

fig. 3: Identifying triangles (Razel & Eylon, 1991, in Sarama & Clements, 2009).

to select triangles than in selecting circles and squares. So, only a few children marked all the correct triangles and only these (cf. tab. 3). About a quarter of the children in both countries selected at the first investigation all triangles and other shapes. At the second investigation, there were clearly less German and about the same amount of English children selecting all triangles and other shapes. The majority of the English as well as the German children marked not all the triangles and other shapes.

Examining which shapes were selected most often, it becomes obvious that most of the children marked the equilateral triangle (no. 8) as such, although it's representation was upside down, immediately followed by the right-angled triangle (no. 1). The "nearly triangles" no. 14 and no. 9 were chosen more often as triangles as the two scalene triangles no. 11 and no. 4.

	Only all the triangles		Only triangles but not all of them		All the triangles and other shapes		Not all triangles and other shapes	
	G	E	G	E	G	E	G	E
2008	5%	6%	9%	26%	28%	24%	56%	44%
2009	12%	9%	26%	29%	14%	26%	44%	35%

tab. 3: Selecting triangles

Altogether, there were more children marking triangles as triangles than children marking other shapes as such. At the beginning of the school year, more German (40%) than English children (15%) marked convex shapes (no. 3 and 7) or the four-sided shape (no. 9) (65% of the German and 47% of the English) as triangles and about half of the children of both countries marked concave shapes (no. 5 and 14) as triangles. At the end of the school year the English children tended to mark slightly more frequently the concave shapes and to the same amount the four-sided shape (no. 9). The children often explained the acute triangles (no. 4 and 11) as being "too pointy", "too thin", "too long" or "too aslope" for a triangle or as in the case of the right-angled triangles as "too straight at the side", "only one long side" or "so high at one side". Altogether, the children often explained in an informal way why certain shapes – actually correct triangles – were not triangles, as for example: "too aslope for a triangle" or "not equilateral". The non-triangles were often justified as triangles by using parts of a correct definition such as: "three sides and three corners" without regarding that the sides were not straight or There also were children, especially English children, who could give a perfect definition of a triangle at a previous task, e.g.: “a triangle has three corners and three straight sides” but were still choosing at the shape selection tasks concave or convex shapes, obviously not having straight sides or selected no. 9 as triangle, which nearly resembles an equilateral triangle.

DISCUSSION

Altogether, the German children, although not formally instructed, more often selected correct circles and correct squares than the English children and about the same amount of children from both countries selected all the correct triangles. However, the English children chose less often wrong shapes as triangles, which shows that they are more familiar with what does not constitute a triangle, presumably due to the instruction in school. The reason why the German children identified circles more correctly is that the English children sometimes simply didn’t mark one or two of the circles, but even the explanations of the children didn’t reveal why they left some correct circles out. Still, what became obvious in the reasoning of the children is that the German children used far more comparisons than the English in order to explain their choices (cf. Maier & Benz, 2012). The English children rather used the names of the shape in order to explain why a shape is or isn’t a circle, e.g.: “this is not a circle because it is an oval” or “that is not a circle, it’s a semi-circle”. Such terms were not familiar to the German children who, for this reason, rather chose comparisons, such as: “this is not a circle, for it looks like an egg”. Why there were far more German than English children at the end of the school year using the properties of a shape to justify their choice does not become obvious through the interviews. Still, other research suggests (e.g. Levenson et al., 2011) that “young children, even those who do not attend a preschool with an especially enriched geometrical environment, (still) employ reasoning with attributes” (ibid. p. 28). This could be the case, because there were now more English children using the correct geometric term and the German children were rather familiar with some properties such as “round” or “acute” than with the geometric term. Another reason for the sometimes different choices of the children could be the material that is used in the single institutions. Consequently, one reason why the English children might only describe a horizontal lying square as a square and one that stands on one of its corners as “a diamond” and not a square anymore, could be the illustrations in the classroom or school books, only showing squares in horizontal position. The reason why they preferred marking isosceles or equilateral triangles as triangles, might be because the material they use for exercising only shows isosceles or equilateral triangles. In order to select correct examples for a certain shape, children need to already have a checklist in their minds what constitutes a certain shape. Otherwise they just choose figures they know, predominantly prototypes. Still, the English children selected more often correct triangles and less often wrong shapes as triangles than the German children, revealing that the Germans were not introduced at this stage to aspects of definitions of a triangle with how a triangle has to look like.

Therefore, the fact that a triangle should have *straight* sides was often not considered by German children.

CONCLUSION

The children of the research seem to conform to a large extent the first van Hiele level (the visual level) or somewhere between the first and the second (descriptive) van Hiele level. Still, the children could be on different levels for different tasks, so the levels seem not to be discrete as previous studies already revealed (e.g. Burger & Shaughnessy, 1986; Clements & Battista, 1992; Lehrer et al, 1998). Furthermore, it can be concluded that the way of teaching as well as the used material are influencing the concept formation of the children. Therefore, it should be discussed *how* to introduce shapes and how to actively support the children's concept formation to develop a comprehensive knowledge about shapes, rather than first clarifying *when* would be the best time to introduce geometric concepts to the children. In order "to know" a shape, being able to identify a wide variety of examples and non-examples is essential (c.f. Levenson et al., 2011). A careful introduction of shapes is important, because research indicates that a lot of educational materials introduce children "to triangles, rectangles and squares overwhelmingly in limited, rigid ways" (Sarama & Clements, 2009, p. 216) as was assumed in this study as well, and moreover that such rigid visual prototypes can rule children's thinking throughout their lives. The visualisation and usage of material in class rooms might be too limited as for often only prototypes are shown. According to Clements (1999, p. 208), such a traditional "prototype-only approach should be rejected". However, the usage of prototypes can be very helpful but teachers as well as kindergarten educators should be aware of the variety of representatives of a certain shape and let them explain what properties a shape needs to have in order to be called "a triangle" for example. An isolated memorising of definitions is to be seen critically and more emphasis should be placed on being able to connect a concept with many representatives as examples. Still, even without being formally instructed but instead informally, children are able to acquire a comprehensive knowledge of concepts.

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