DIFFERENCES BETWEEN BEGINNERS' AND EXPERIENCED STUDENTS' APPROACHES TO LEARNING MATHEMATICS AT THE UNIVERSITY

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In this paper, we report on an on-going quantitative study of students' transition from school to university mathematics. The study aims at examining differences between beginners and experienced students' approaches to learning mathematics. Students were given questionnaires in the beginning and at the end of their first year at university. The results were summarized with descriptive and interferential statistics. The results show that beginners rely heavily on the teacher, while experienced students re-orient themselves from the teacher to other kinds of mathematical resources, for example peers and Internet based resources.

INTRODUCTION

In this paper, we report on the first part of an on-going quantitative study of mathematics students in transition between secondary and tertiary level. We focus their experiences of mathematics studies from secondary school and after one semester of their university studies. In particular, we are interested in the changes that take place during the students' first courses in mathematics at the university. The aim with this study is to examine students' approaches to learning at the beginning of their mathematics studies at the university and after their first mathematics courses when they are more experienced as university students in mathematics. After a short report of previous research that is related to our study, we give an account for the conceptual framework that has been used in our study. The results from descriptive and inferential statistics are discussed in the final section of the paper.

Previous research

Despite extensive efforts to make the transitional phase from school to university mathematics easier, entering higher studies in mathematics still seems to cause problems for many students (Gueudet, 2008). Entering university studies in mathematics put new demands on novice students' adaptability, both to a partly new mathematical content, their beliefs and values and to a new learning environment (Wood, 2001). Previous experiences of mathematics education from secondary level can have a crucial impact on the transition, for example if the pace of study is incresignifically higher or if the relation to the teacher changes (de Guzmán et al., 1998). This may call for changes of students' approaches to learning and view on knowledge (Perry, 1970), which seem to be a crucial step for students to successfully undertake the transition into university mathematics (Stadler, 2009). Attempts have been done to develop quantitative measuring instruments to find correlations between students' view of knowledge and their learning approaches (Kemper & Leung, 1998) as well as students perception of changes when learning mathematics when entering university and their mathematics disposition (Pampaka, Williams & Hutcheson, 2011). Also, the relation between affective variables such as students' beliefs and self-conception and students' success in the transitional phase from school to university mathematics have been examined, where a correlation between students' beliefs about mathematics and their approaches to learning has been shown (Liston & O'Donoghue, 2009).

In this study, we will focus on differences between beginners and more experienced mathematics students. Novice mathematics students at university have various experiences from mathematics teaching and learning from secondary level, which in turn influence their approaches to learning mathematics (Stadler, Bengmark, Thunberg & Winberg, 2012). Moreover, to have been subject to mathematics teaching is crucial for students' transformation from novice to expert mathematical problem solvers and their approaches to work mathematically (Schoenfeld, 1982). Consequently, we assume that students' approaches to learning are likely to change during the transition into university studies. The aim with our study is to examine students' approaches to learning mathematics with focus on differences between beginners and experienced mathematics students at the university.

CONCEPTUAL FRAMEWORK

In this section, we present the concepts that we have chosen to use in our study to examine *students' approaches to learning mathematics*. In a qualitative study of students' transition from secondary to university mathematics, Stadler (2009) identified three concepts that describe students' approaches to learning mathematics.

The *mathematical learning objects* category refers to the students' view of the overall purpose of learning mathematics. It captures students' interpretation of what mathematics is and what learning mathematics is all about. Students' subject specific beliefs about learning and knowledge have been shown to strongly relate to their reasoning ability in the subject (Winberg & Berg, 2007). Perry's scheme (1970), modified by Winberg (2006), describes stages of college students' intellectual and ethical development. It is regarded as a continuum between two views of knowledge and learning; from an absolute and transferable knowledge, which can be either right or wrong, to a relativistic view of knowledge, which is contextually dependent and where the students take responsibility for their learning. If university teachers teach mathematics in a way that corresponds to a higher level in the scheme, students can regard the teacher less useful (Stadler, 2009). Thus, from a student perspective, the transition brings about a need to re-orientate towards new and modified mathematical learning objects compared to secondary school.

Mathematical resources are objects and phenomena that students use to learn mathematics. Some examples are the textbook, the teacher, the peers, the students' pre-knowledge in mathematics and their logical thinking. For novice students in mathematics, modifications of the use of mathematical resources are an essential part of the transition (Stadler, 2009).

Students' actions as learners are closely related to their goals and aims of learning mathematics, are contextually dependent and may vary over time. Differences between mathematics teaching in secondary school and university call for new ways of approaching learning and actions to learn. A distinction can be made between independent and dependent actions as learners, where the former indicates that the students undertake actions they chose by themselves, whereas the latter means that even though students have intentions, they are not always able to undertake those actions that they find necessary to accomplish their intentions.

Choosing these concepts to examine students' approaches to learning is a methodological approach that combines qualitative and quantitative research methods (Winberg, 2006). In the qualitative study, crucial aspects of learning mathematics for students in transition were identified. The quantitative study advances these positions by examining which aspects are more crucial than others, to what extent and for whom. For example, the qualitative study shows that the use of partly new mathematical resources but also to use familiar mathematical resources in a slightly new way is important for mathematics students in transition (Stadler, 2009). The quantitative study gives supplementary information about to what extent new and other mathematical resources are used and by which category of students.

METHOD

We have developed a research instrument with two questionnaires consisting of 13 query themes were designed and implemented (table 1). The choice of themes was based on the conceptual framework of students' approaches to learning and their beliefs, motivation and self-concept regarding mathematics, as presented previously in the paper. The first questionnaire focused the students' previous experiences of studying mathematics at secondary level and their expectations on mathematics studies at tertiary level. In the second questionnaire, almost the same questions were asked but now with focus on their mathematics studies at the university after approximately one semester of mathematics courses.

In our study, we have chosen not to pose explicit questions about the students' mathematical knowledge. Instead, we have collected data about their previous grades from secondary level and study results on their initial mathematics courses at the university. Students' preferences about mathematical resources were captured by questions about their evaluation of different mathematical recourses that turned out to be crucial for the transition (Stadler, 2009) and behaviour in relation to these mathematical resources. Thus, we have asked question about what the students actually do when they aim at learning mathematics. To examine which mathematical learning objects the students are focusing, we have examined students' orientations towards mathematics and the learning of mathematics.

Questionnaire 1	Questionnaire 2
1. Entry requirements. 2. Lesson activities in upper secondary school. 3. Valuation of lesson activities in upper secondary school. 4. Help-seeking behaviour during mathematics lessons in upper secondary school. 5. Valuation of homework activities outside school. 6. Help-seeking behaviour during homework in upper secondary school. 7. Valuation of resources for the learning of mathematics. 8. Valuation of the mathematics teacher's actions 9. Valuation of working with peers. 10. Valuation of the textbook. 11. Orientations towards mathematics and the learning of mathematics. 12. Expectations concerning forthcoming studies of mathematics at university. 13. Expected requirements for succeed with mathematics studies at university.	1. Entry requirements. 2. Lesson activities at university. 3. Valuation of lesson activities at university. 4. Help-seeking behaviour during mathematics lessons at university. 5. Valuation of homework activities outside school. 6. Help-seeking behaviour during homework at university. 7. Valuation of resources for the learning of mathematics. 8. Valuation of the mathematics teacher's actions. 9. Valuation of working with peers. 10. Valuation of the textbook. 11. Orientations towards mathematics and the learning of mathematics. 12. Experiences of mathematics studies at university. 13. Experienced requirements for succeed with mathematics studies at university.

Table 1: Query themes in the questionnaires

Questions 3 to 13 were formulated as Likert scale questions with a five-step rating scale. For example, the initial questions about beliefs and attitudes towards mathematics and the learning of mathematics were formulated as follows:

11. Here are some questions about your views of mathematics and learning of mathematics.

	Strongly Disagree				Strongly Agree
a) It´s easy for me to learn mathematics.	1	2	3	4	5
b) I can solve most exercises by myself.	1	2	3	4	5
⁝					
w) I learn new concepts by solving exercises.	1	2	3	4	5

The participating students came from different universities and various study programmes. However, in this report, we have chosen not do discriminate between specific groups of students (see Stadler, Bengmark, Thunberg & Winberg, 2011). All students were chosen according to availability. In total, 146 students answered the first questionnaire while 134 students answered the second questionnaire. Both questionnaires were distributed to the same groups of students.

The first questionnaire was distributed on mathematics lectures during the first two weeks of the first semester of the study program. At the university, the second questionnaire was distributed at the end of the first semester. At the two technical universities, the second questionnaire was distributed in the middle of the second semester. The different times of implementation were due to organisational differences of the mathematics courses at each university. However, all the participating students had studied at least two university mathematics courses at the time for the second questionnaire. Each questionnaire took 15-25 minutes to answer.

The quantitative data have been analysed using two methods. Firstly, we have used *descriptive statistics* to summarize data in order to describe the main features of the participating students and to be able to compare the results from the first and second questionnaire. Secondly, *inferential statistics* with discriminant analysis with PLS (PLS-DA), a regression extension of Principal component analysis, was used to describe the relative importance of questionnaire items for discriminating between the beginner- and experienced student group, that is, to investigate the distinguishing features of these two groups of students. The data from the first questionnaire has been separately analysed and reported on in a previous paper (Stadler et al., in press).

RESULTS AND ANALYSIS

With the descriptive statistics, we focus on changes of students' characteristics as learners of mathematics. For each Likert scale question we have calculated the mean value for beginners and experienced students and used a two-sided t-test to investigate possible significant differences between the two groups of students. In the first questionnaire the students' previous experiences of mathematics studies at secondary level were examined (Stadler et al., in press). According to the students, a typical mathematics lesson begun with the teacher giving a short introductory lecture, which lasted for 10-15 minutes. The rest of the lesson, the students worked with textbook exercises. In the questionnaires, the beginner and the experienced students were asked about the importance of the following lesson activities for their learning of mathematics at secondary and university level respectively (table 2).

	Beginner	Experienced	95% CI for difference	p
Lectures	4.24	3.99	[0.01; 0.49]	.04
Demonstration		4.36	[-0.34; 0.09]	.26
Individual work with exercises	4.28	3.99	[0.05; 0.53]	.02
Individual help from teacher	3.92	3.07	[0.55; 1.16]	.00
Exercises with peers	3.37	4.02	[-0.96; -0.35]	.00
Discussions with peers	3.20	4.13	[-1.21; -0.65]	.00
Internet based resources	0.94	2.56	[-1.93; -1.30]	.00

Table 2: Evaluation of lesson activities, query theme 3.

The beginners value two kinds of lesson activities as the most important, namely lectures and demonstrations from the teachers, and work with exercises. The focus seems to be on working with exercises with the support of a teacher. The experienced students give higher ranking to peers, while the main contribution from the lectures are demonstrations of how to solve exercises. The shift from relying on the teachers to the peers is in accordance with Stadler's findings that the students in transition are forced to an increased independency and autonomy (2009). Another difference between beginners and experienced students is the use of Internet based resource.

The students' valuation of the importance of various mathematical resources is shown in table 3.

	Beginner	Experienced	95% CI for difference	p
Teacher	4.29	4.16	[-0.11; 0.36]	.29
Peers	3.74	4.23	[-0.73; -0.26]	.00
Textbook	4.05	4.22	[-0.38; 0.04]	.11
Previous tests	3.20	4.25	[-1.29; -0.82]	.00
Book of formulas	3.60	3.04	[0.28; 0.84]	.00
Calculator	3.43	2.14	[1.04; 1.52]	.00
Computer for calculations	1.33	2.42	[-1.32; -0.87]	.00
Internet based resources	1.34	2.56	[-1.45; -0.98]	.00

Table 3: Evaluation of mathematical resources, query theme 7.

The beginners' evaluation of mathematical resources are in tune with the mathematics education that they have experienced at secondary level; the teacher gives a short introduction, which is followed by individual work with textbook exercises where the students are allowed to use the book of formulae and graphic calculators as resources. The appraisement of mathematical resources changes in favour for previous written examinations, peers, computers and Internet based resources when the students become more experienced. According to Stadler (2009) the teacher as a mathematical resource changes at the university. Instead of giving instructions of how to solve exercises, the focus is on general mathematical ideas. To novice students, the information from the teacher becomes less useful. However, examples, previous tests and the peers can still provide information and instructions of how to solve exercises.

The students' beliefs about what will be or are the most important things to do to succeed with their mathematics studies at the university are shown in table 4.

	Beginner	Experienced	95% CI for difference	p
Attend all lectures	4.32	3.72	[0.37; 0.84]	.00
Study outside school	4.29	3.55	[0.50; 0.97]	.00
Do as the teacher tells you to	4.24	3.61	[0.40; 0.86]	.00
Study theory in the textbook	4.23	3.92	[0.09; 0.52]	.01
Solve many exercises	4.18	3.90	[0.05; 0.51]	.02
Attend all tutorials	4.14	3.86	[0.05; 0.53]	.02
Get help from the teacher	4.05	3.12	[0.67; 1.9]	.00
Get help from the peers	4.03	4.02	[-0.21; 0.23]	.93

Table 4: Valuation of study activities, query theme 13.

Worth noticing is that the experienced students value the importance of all study activities lower than the beginners, except for the peers. The importance of the teacher for the students' study success decreases and study activities decrease in importance. These results indicate that the students to a greater extent valuate manage their studies on their own or with peers.

The students' actions as learners can be categorized as dependent or independent (Stadler, 2009). This is a crucial aspect of the transition because the students are to a greater extent forced to manage their studies on their own, relying more their ability to read and learn and to use peers and resources on the Internet. Table 5 shows the differences between beginners' and experienced students' help seeking behaviour.

	Beginner	Experienced	95% CI for difference	p
Teacher	3.60	3.02	[0.35; 0.80]	.00
Peers	3.67	4.04	[-0.57; -0.16]	.00
Reading theory	3.61	3.81	[-0.42; 0.03]	.10
Solve examples	3.79	4.18	[-0.58; -0.19]	.00
Study notes from lectures	3.02	3.34	[-0.61; -0.02]	.04
Other peers' solutions	2.63	2.99	[-0.61; -0.12]	.00
Using the Internet	1.43	2.72	[-1.54; -1.04]	.00
Skip the exercise	2.33	2.56	[-0.43; -0.02]	.03

Table 5: Help seeking behaviour, query theme 4 and 6.

While students' help seeking from the teacher decreases, peers and the Internet are two interactive mathematical resources that increase in importance. The textbook, other notes and solutions of exercises are also available mathematical resources that can be used anywhere and anytime. Also, the intensity in the interaction between the teacher and the students usually increases at the university (Stadler, 2009).

In contrast to descriptive statistics, inferential statistics can be used to find correlation patterns in data and to find out the relative importance of different variables. We have performed a PLS discriminant analysis to further discern differences between beginners and experienced students. We generated a model with three significant components, according to the cross validation procedure. However, as the first component was able to predict 65 % of the variation in group belongingness and the second and last component 13 % and 8 %, respectively, only the two first components are presented in the loading plot (figure 1). Items that are close to the experienced group, in particular in the horizontal direction, describe features that are typical for the experienced students and a-typical for the beginner students. The opposite is true for items that are closed to the beginners group.

The interpretation of the loading plot and an analysis of variable importance for projection (VIP) revealed the most important features that discerned experienced students from beginners. Experienced students perceived a lower usefulness of calculator (item: 7l; VIP: 2.4) and an increased perceived value of internet-based resources to ask questions and find answers (7j; 2.2) and actual use of internet-based resources to seek information to support learning and problem solving in school (4g;

2.2, 3h; 2.2) as a calculating tool or task bank (7i; 2.1, 5e; 1.8). Internet based resources was also perceived valuable as general support (5f; 2.1) or problem solving assistance (6g; 2.0) when studying math outside school. However, the computer as a calculator was not considered as important for study success (13i; 1.3).

Furthermore experienced students, to a greater extent than beginners, view peers as an important resource, for joint problem solving (9b; 2.1) discussion of theory and concepts (9e; 1.9) and question asking (9d; 1.9, 6b; 1.6) outside school. Getting help from the teacher (13l; 1.6 3b; 1.3), as well as preparing well before going to the lectures (13a; 1.6), daily work with the course after school, doing what the teacher tells them (13e; 1.3), and attending all lectures (13b; 1.2) were considered less important for study success by experienced students than beginners.

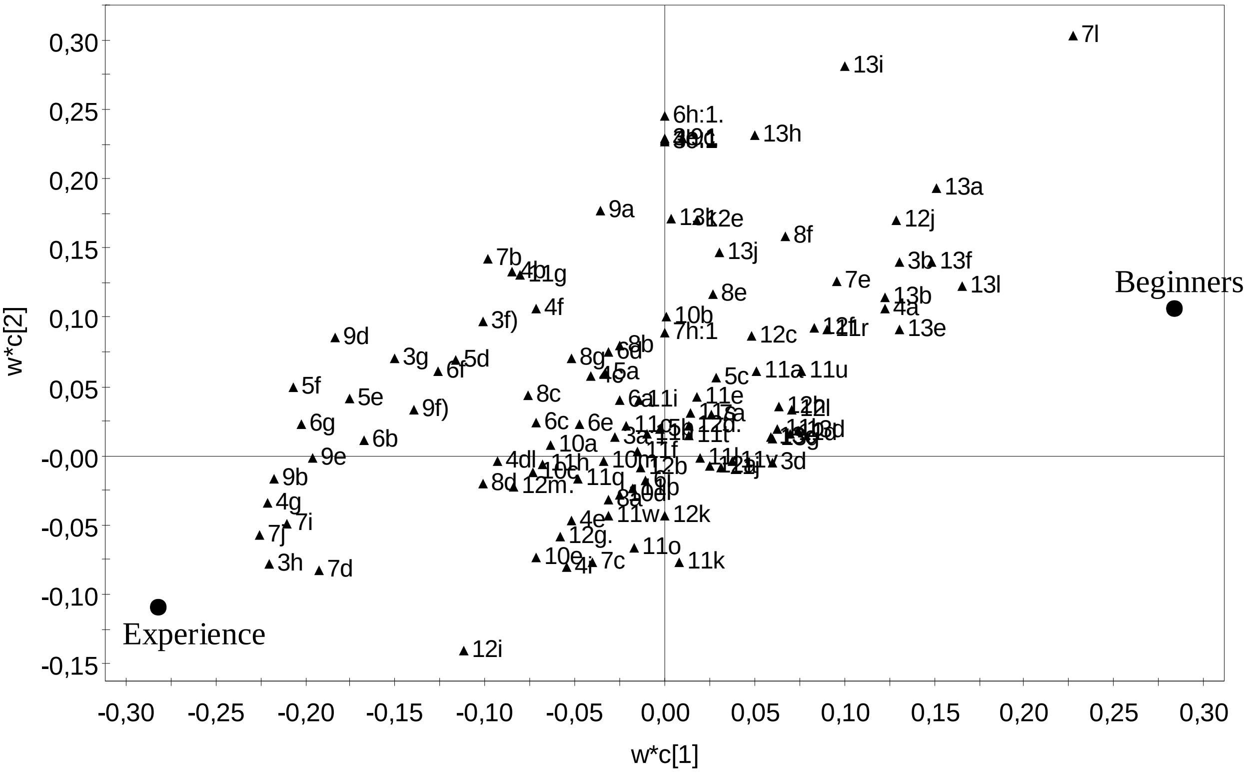

Figure 1: Loading plot from the PLS-DA

The loading plot from the PLS-DA graphically shows the relative importance of the unique items to describe the distinguishing features of beginners and experienced students respectively. 80 % of the variation in group belongingness is described (R2) and 72% predicted (Q2) by the model, using 22 of the total variation in the predicting variables (i.e. the unique questionnaire items).

The validation of the model has been made through:

- Hotellings T2 range: one student was beyond the 99% level. On deletion of this student, no substantial differences in the model occurred (VIP:s, correlation patterns and R2 and Q2 were virtually identical)

- DmodX (distance to model): no outliers were detected (eg. having DmodX values exceeding 2x crit 95% limit)
- Response permutation testing: R2 and Q2 were significantly lower than for the original model when Y values were permutated, meaning that the models predictions were not spurious/by chance.
- Observation risk – no observations (i.e. students) displayed critically high residuals (Orisk above 1.5). Hence, there was no single student who overly affected the model predictions of group belonging (e.g. by being far from the centre in X-space and having large residual when not being part of the training set, compared to when it was)

DISCUSSION AND CONCLUDING REMARKS

The aim with this study was to examine students' approaches to learning mathematics with focus on differences between beginners and experienced mathematics students at the university. The results indicate that during the transition, the students' approaches to learning changes. This may be due to the students' exposition to university teaching of mathematics (Schoenfeld, 1982) that may differ from their previous experiences of mathematics education from secondary level (de Guzmán et al., 1998). Shifting focus from the teacher to the peers can be interpreted as a way do adapt to a new learning environment, which is a curial ingredients of the transition (Wood, 2001). The decreasing significance of the teacher be due to limited availability of the teacher or that the university students found the explanations and help from the teacher less useful (Stadler, 2009). However, a higher value might have been predicted since the university teacher can be regarded as the best representative for the new epistemological beliefs and approaches to mathematics that the students have to adapt to in the transition. The decreasing values of teacher and the increasing value for peers may be interpreted as the students attempting to handle the transition without changing their epistemological beliefs and approaches to mathematics.

Despite that the multifaceted transition cause problems for the research area, our approach has rather been to take the complexity as a starting point and yield a result that shows which variables are the most crucial for students in transition. Previous studies have mainly adopted a bivariate approach to predict the outcome (Pampaka et al., 2011; Kemper & Leung, 1998), which limits the possibility to understand the impact of confounding variables and yields a lower predictability than multi variant methods. Compared to other previous studies (Pampaka et al., 2011) we have also included questions about students' actions and behaviour to be able to correlate them, their performance and their beliefs and experiences.

Our results indicate important insights about students in transition. However, we are aware of the danger of jumping to too far-reaching conclusions. The sample is small and not representative for all mathematics students in transition. We also lack the information about whether individual students have answered both the first and the second questionnaire. The results that have been presented in this paper give an indication of what can be the crucial differences between beginners and experienced mathematics students. Also, the results can be used as an indication of what to expect from a larger sample and how to design forthcoming studies.

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