CAPTURING PRE-SERVICE TEACHERS' MATHEMATICAL KNOWLEDGE FOR TEACHING OF GEOMETRY

Ravi B. Somayajulu

Eastern Illinois University, Charleston, USA

This study investigated the change in the Mathematical Knowledge for the Teaching (MKT) of geometry for three preservice teachers over a period of seven months. Data was collected via pre- and post-surveys as well as interviews. Data collection items were specifically designed to elicit preservice teachers' MKT as they responded to items involving the analysis of student work and thinking. The results indicated that there was a significant decrease in preservice teachers' scores on items pertaining to analysis of student work and thinking. Preservice teachers relied on their past experiences as students and on their work experience while making pedagogical decisions. Recommendations for future research in preservice teacher education are discussed.

Keywords: Preservice Teacher Education, Mathematical Knowledge for Teaching, Geometry

INTRODUCTION

Until a few years ago, researchers in the U. S. defined teachers' subject matter knowledge using quantitative means such as number of subject specific courses taken or the teachers' scores on standardized tests (Even, 1993). However, the adequacy of these proxies for measuring mathematical knowledge for teaching has been the subject of controversy and debate (Hill, Rowan, & Ball, 2005). Ball et al. (2005) coined the term Mathematical Knowledge for Teaching (MKT) in order to distinguish a specialized body of knowledge of subject matter that is needed for teaching. MKT consists of not only knowledge of mathematics content, but also how that content is taught. It includes knowledge about which topics or concepts are easy for students to learn and which of them are difficult. While several studies at the elementary and middle school level have claimed that teachers' MKT is correlated to student achievement, such studies are rare at the secondary school level. One of the reasons for this is that high school mathematics consists of a variety of topics and is very complex (McCrory, Ferrini-Mundy, Floden, Reckase, & Senk, 2010). Knowledge about how secondary students think about mathematics is also very limited.

This work is a part of my dissertation study in which I investigated the nature of MKT of secondary preservice teachers by eliciting their reactions to samples of student work included in written surveys, using a case study methodology (Somayajulu, 2012). The following research question guided the data collection and analysis:

> What factors do preservice teachers consider when making pedagogical decisions based on analysis students' mathematical work and thinking?

THEORETICAL FRAMEWORK

MKT has been given several definitions, but i subscribe it to being described as "mathematical knowledge needed to carry out the work of teaching mathematics" (Ball et al., 2008, p. 395). For the purpose of this study I utilized the Knowledge for Algebra Teaching (KAT) (McCrory et al., 2010, 2012). Although the KAT framework has been designed for assessing algebra teaching, the teaching tasks and the knowledge categories that are highlighted by the framework are applicable to all the areas of mathematics, and in particular to geometry. This framework consists of a two dimensional matrix in which the rows represent the tasks that teachers perform while teaching mathematics and the columns represent the categories of knowledge required to perform those tasks. The specific categories can be seen in detail in Figure 1 below. For the purposes of the current study, I focused on the task of analysing student work and thinking and the knowledge bases required to successfully perform that task.

In addition to the tasks of teaching and the categories of knowledge, the framework also consists of three overarching categories: decompressing, bridging and trimming (McCrory et al., 2010). Decompressing, according to McCrory et al. (2010) is working from a more compressed understanding of mathematics to a more unsophisticated form. Decompressing includes "attaching meaning to symbols and algorithms that are typically employed by sophisticated mathematics users in automatic, unconscious ways" (McCrory et al., 2010, p. 38). Trimming is described as a process in which teachers present an advanced or sophisticated mathematical idea to students in a way that the fundamental nature of the topic is preserved but it is now less rigorous (McCrory et al., 2012). Bridging involves making connections between mathematical topics or between mathematics and other subject areas (McCrory et al., 2012). Bridging is similar to the actions that teachers need to perform, which are highlighted by Ball et al (2008) in their category of Knowledge of Content and Teaching (KCT). The KAT framework guided the task selection for the surveys.

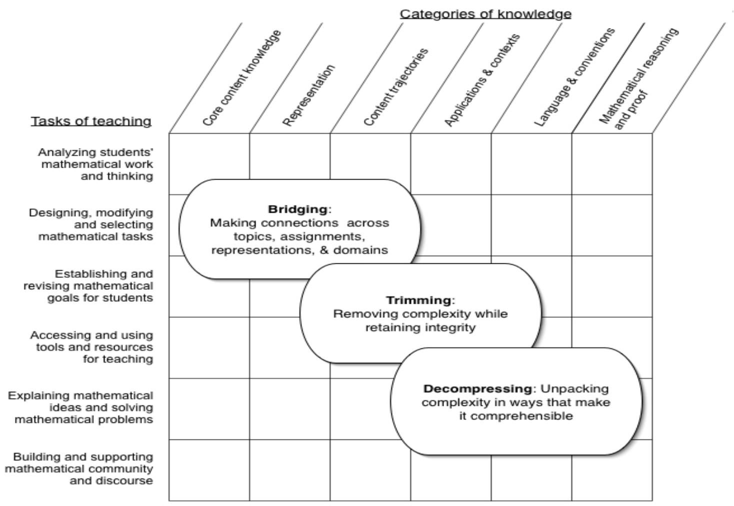

Figure 1: Knowledge for Algebra Teaching (McCrory et al., 2010, p. 58)

METHODOLOGY

Participants

The original dissertation study involved of eight participants: four male and four female enrolled in the Master of Education program seeking licensure to teach 7th to 12th grade mathematics in the U.S. Six of the participants had a bachelor's degree in mathematics while two of the participants had a background in engineering. For the purpose of this study, I report on in-depth a case study involving three of the participants. The selection of these three participants was done based on Cooney, Shealy and Arvold's (1998) classification of preservice teachers. A brief description of the three case study participants is given below.

Cersei

Cersei had a background in industrial engineering prior to joining the M. Ed. Program. Cersei fell in the isolationist category (Cooney et al., 1998) in that she rejected the beliefs of others. The program had very little effect on Cersei's beliefs about mathematics teaching and learning.

Bran

Bran completed his bachelor's degree in mathematics prior to joining the program. Bran demonstrated the qualities of a naive connectionist (Cooney et al., 1998) in that he was receptive to other's beliefs but is not able to resolve the conflicts between his beliefs and other's beliefs.

Nedd

Nedd was a computer engineer before joining the program. Nedd was a naive idealist (Cooney et al., 1998). He readily accepted other's views and beliefs without questioning them. Unlike the other participants, Nedd opted for an extra year to complete the program.

Data Collection and Instruments

Data collection took place over a period of seven months via pre and post surveys and interviews. The surveys and interviews were designed to capture the preservice teachers' MKT as it pertained to the analysis of student work and thinking via the processes of decompressing, trimming, and bridging. The survey items were taken from previously recorded episodes of student work on geometric tasks consisting of instances of children's thinking and heuristic usage (Manouchehri, 2012). Additionally survey items were chosen from mathematical topics that are common to the U. S. secondary school curriculum. The following is an example of one chosen task:

> A student was given the following problem: Consider a cube whose base area is 4 cm^2. If the area of the base increases to 16 cm^2, how much does the volume increase?
>
> The student replies by saying that the volume of the cube would increase by 4096 cm^3. How do you think the child arrived at this answer?

What techniques or tools may be used to help the child understand the solution?

What are some questions you can ask the student to further his understanding on the topic?

The content validity of the survey instrument was verified by obtaining feedback from practicing teachers, mathematics educators, and mathematics education graduate students. The surveys were piloted with graduate students in mathematics education and based on their responses and feedback, necessary modifications were made. Finally the surveys were administered to preservice teachers outside of the sample for this study. Based on their responses final changes were made to the language in order to remove any ambiguities.

Interviews were conducted after the administration of the surveys to gather further information of the participants' responses to the surveys. The interviews were approximately 90 minutes long. The interviews consisted of three parts: (1) obtaining background information from each participant, (2) a self efficacy scale (Tschannen-Moran, & Hoy, 2001), and (3) focused on exploration of survey responses.

Data Analysis

The data analysis was completed in two phases. During the first phase, the preservice teachers’ responses to the surveys were analysed and coded. In order to do this, the survey responses were classified along two categories of mathematical and pedagogical analysis. I utilized previously identified performance indicators from a study exploring knowledge for teaching (Manouchehri, 2011). The indicators are as follows:

Mathematical Analysis

Articulating basis for mathematical decisions that children make.

Identifying the strengths and weaknesses of ideas from a mathematical point of view.

Identifying content trajectories and using them to assess children’s conceptions.

Identifying the sources of children’s errors/misconceptions and reasoning how they could have resulted.

Developing mathematically sound instructional strategies.

Pedagogical Analysis

Identifying why certain pedagogical moves are appropriate to pursue with children based on their analysis of student work and thinking

Successfully addressing areas in which children would or would not be able to perform adequately.

Offer a rationale for why certain pedagogical choices should be implemented.

Identifying the advantages and disadvantages of use of instructional tools.

Based on these indicators, I categorized the responses on a continuum from Mathematically and Pedagogically Naïve to Mathematically and Pedagogically Mature. The scoring of the responses is explained in table 1. This resulted in a maximum possible score of 4 and a minimum possible score of 0.

Score	Mathematical Analysis	Pedagogical Analysis
0	Mathematically Naïve	Pedagogically Naïve
1	Mathematically Developing	Pedagogically Developing
2	Mathematically Mature	Pedagogically Mature

Table 1: Scoring Rubric

Since the number of prompts used differed from the pretest to the posttest, I calculated a percentage score for each participant (that is total points scored divided by total points attempted on a particular survey). These scores from the pre and posttest surveys were then compared using a paired-samples t-test for the entire cohort. In the event that the question was not answered, it was not scored and left out of the analysis.

The second phase of data analysis involved analysing the interview data. The interviews were transcribed and coded based on the references they made while analysing student work. Once the interviews were coded, percentages of occurrences for each of the coded category were calculated. Those percentage scores were utilized to build an illustrative map of sources that the preservice teachers drew from while analysing student work and thinking. Finally interview data was compared with the responses to the surveys in order to triangulate conclusions. An example of an illustrative map is given below. The map describes the various factors that Cersei referred to while analyzing student work and thinking during the pretest interview.

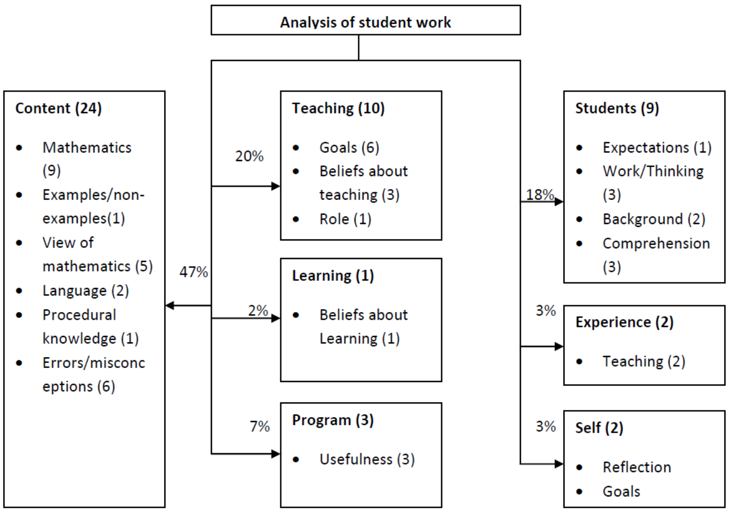

Figure 2: Example of an illustrative map highlighting factors influencing participants decision making

From the above example we see that Cersei made a total of 51 references to student work in her interview. While commenting on student work, Cersei referenced the mathematical content 24 times and teaching 10 times. She also made 9 references to students. Other references were made to learning, experiences, M. Ed. Program and to self. These categories were further broken down into subcategories. So for example, while commenting on the content, Cersei referenced the mathematics 9 times, while she referred to the child's errors/misconceptions about 6 times.

Such maps were generated for both pretest and posttest for each of the three case study participants. These maps were then used to generate a list of important factors that the preservice teachers consider while making pedagogical decisions.

RESULTS

The findings for each of the three case study participants are discussed below.

Cersei

The major factors affecting Cersei's orientation toward teaching were her experiences both as a learner of the subject as well as her work experience, and her beliefs about the teaching and learning of mathematics. She viewed mathematics teaching as being similar to her job as an industrial engineer. When asked if being an engineer was different/similar to being a teacher she replied:

> Cersei: I was always the person that trained the new industrial engineers coming into the department. So I guess I've kind of been teaching all along without really recognizing it but really liked in that whole experience just to being kind of a mentor and helping so many get started into the department.

Analysis of Cersei's responses to the surveys revealed that there was a decrease in her scores for prompts pertaining to analysis of student work and thinking, decompressing, trimming and bridging. After augmenting her interview data with the survey data it was observed that even though her attention to student work had increased, there was a decrease in her attention to the mathematical content. Cersei was also not able to make specific connections to models of assessment and hence was unable to utilize them while analysing student work and thinking.

Bran

The main factors affecting Bran's orientation towards teaching were his experiences as a learner of mathematics as well as his knowledge of the subject matter. Bran acknowledged the connections between his college mathematics courses and high school mathematics and drew on this knowledge of content trajectories while attending to student work. For example, one of the questions on the survey asked the participants to choose two of three topics (similarity, transformations and right triangle trigonometry) such that it would foster student learning. Bran selected transformations and right triangle trigonometry. He did not choose similarity because he felt that it could be derived from transformations.

Researcher: You said similarity can be described through transformations, and how would you go about that?

Bran: I think I'd start with a shape. So a transformation of this would be to make each side longer. So I could keep this here, so I can make everything, like move these points twice as far away from each other. And so I could have two shapes that after the transformation, the shape is still similar. So I can tell that's a special case of transformation when the relationships between the sides and the angles are the same.

Comparison of Bran’s pre and posttest survey scores revealed that there was an increase in his scores for prompts pertaining to the analysis of student work and thinking, trimming, and bridging. However, there was a decrease in his scores for prompts pertaining to decompressing. His interview data revealed that his attention to the mathematical content had increased. Bran also demonstrated an increased tendency to try and understand the reasoning behind the student work.

Nedd

Work experience was the biggest factor affecting Nedd’s orientation to teaching. Nedd was of the opinion that his job as an engineer had already instilled in him the skills required to be a successful teacher. When I asked Nedd if being a teacher was similar to being an engineer he replied that they required the same set of skills.

Nedd: Yeah. So I think the biggest reason, the biggest thing is, umm, mostly continuous improvement. You know problem solving, adaptability, looking at all the inputs. You know, defining a problem, alternatives, what gets you to that next improvement. I mean, I think those are very drilled into me and I think that’s a skill a teacher needs to have.

Nedd also showed a decrease in his scores pertaining to analysis of student work and thinking, decompressing and bridging. From the interview data, I observed that Nedd’s attention to student work decreased. Instead the tasks pertaining to student work and thinking served as an avenue for Nedd to reflect on his own mathematical knowledge.

A cross examination of the three cases led to the following findings:

Preservice teachers had trouble decompressing their knowledge while analyzing student work and thinking.

Preservice teachers relied on their experiences while analysing student work and making pedagogical decisions.

Preservice teachers were unable to utilize learning based assessment models such as van Hiele or Pirie-Kieren (Pirie, & Kieren, 1994) to aid in evaluating student work as well as designing instructional tasks.

Knowledge of content and content trajectory had an impact on the participants’ analysis of student work and their pedagogical decision-making.

DISCUSSION

Figure 3 illustrates the common factors affecting the preservice teachers' pedagogical decision making. Participants' past experiences along with their beliefs about the teaching and learning of mathematics, and their knowledge of the trajectory of the content were critical factors when making pedagogical decisions while analyzing student work and thinking.

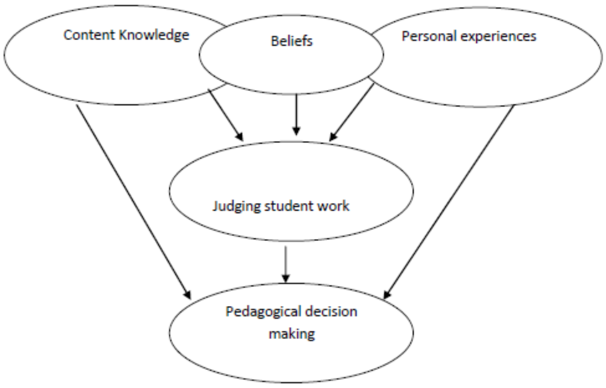

Figure 3: Forces influencing pedagogical decision making (Somayajulu, 2012, p. 282)

Elbaz (1983) demonstrated the interactions between teachers' personal theories and practice, often referred to as teachers' personal theorizing. This study exhibited similar results, where in the preservice teachers relied on their past experiences and their classroom observations of their mentor teachers while making pedagogical decisions. Of these, their past experiences as learners of mathematics and their work experiences had a prominent influence on their decision making. While referring to student work, the preservice teachers almost always made decisions independent of attention to student work and relying mostly on their experience as learners of mathematics. For the participants with engineering backgrounds, work experience was a constant influence on how they viewed the mathematical content.

Several studies have demonstrated the importance of the knowledge of content and its trajectory while making decisions (Aubrey, 1996; Kahan, Cooper, & Bethea, 2003). This study too demonstrated that such knowledge was critical in pedagogical decision making. Preservice teachers, who were not comfortable with the content, were not able to identify the concepts that were central to the topic and hence could not identify the trajectory of contents.

One major concern was that none of the participants were able to connect theory to practice. This concern has been documented by several researchers, in particular by Jaworski (2006). According to Jaworski (2006), this inability to connect theory to practice stems from the fact that even though theories are valuable tools for analysing student work, they do not offer any clear insights to teaching. I observed that none of the preservice teachers in the study were able to apply models of assessment to aid them in their analysis of student work and thinking. One reason for this is that it is unclear how we can measure teachers' development within the realm of a research based program. It is essential to develop an understanding of teachers' learning trajectories as they are exposed to new knowledge via teacher education programs.

LIMITATIONS

A major limitation of this study was that the analysis was solely based on the results of the survey and the interviews. While I observed the participants in their methods courses, I did not analyse the discourse in those classrooms. Such analysis would have helped in providing deeper explanations of why certain changes took place.

The survey instrument that was developed covered a limited range of topics. Moreover the surveys were about 150 minutes long and as a result some of the participants left items blank, especially at the end of the posttest surveys.

RECOMMENDATIONS

There is a need for developing instruments that are capable of capturing the different aspects of MKT. Such instruments also need to cover a broader range of topics. While doing so, steps need to be taken so that the surveys are not too long. Another recommendation for future would be to consider the use of interviews as venues for learning and growth of preservice teachers.

Studies analysing classroom discourse and interactions amongst preservice teachers need further attention as they have potential to offer perspectives on how to sequence tasks to better aid in teacher preparation. There is a further need to analyze the interaction between content knowledge and teaching at the secondary school level.

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