MATHEMATICAL CREATIVITY AND HIGHLY ABLE STUDENTS: WHAT CAN TEACHERS DO? AN ANALYSIS OF DIDACTICAL REGULATIONS OF DIFFERENT COGNITIVE CAPACITIES IN TEACHING AND LEARNING RELATIONAL CALCULATION TO 9-10-YEAR-OLD STUDENTS

Bernard Sarrazy[1)], Jarmila Novotná[2)]
Laboratoire Culture, Education, Sociétés (LACES EA4140)
[1)] Université Bordeaux Segalen (France), [2)] Charles University in Prague (Czech Republic)

The text focuses on the effects of didactical heterogeneisation on students' creativity, namely students' ability to come up with new solutions to problems that are novel to them. The fundamental questions for mathematics education are: Should the teacher orient his/her teaching towards good mastery of algorithms or towards development of students' creativity? Should all students or only the highly able be given the opportunity to work creatively?

Key Words: Novel solutions, students' creativity, teaching strategies

INTRODUCTION

The question addressed by this text might be formulated simply: If learning mathematics consists of coming up with new solutions to new problems (in the student's perspective) and not only of mere reproduction of algorithms, then it is mathematical creativity which is in the center of mathematics education; some students allow themselves to create new solution to a larger extent than others. Who are these students and can they (often termed "intelligent", "highly able" or "gifted") be regarded as extra load in the teacher's work (this attitude can be come across in some cases)? It must be taken into account that it may not be possible to treat these differences in talent by the teacher from the point of view of didactics. This is the thesis that we will defend here.

When Alfred Binet (1857-1911), the French psychologist and inventor of the first usable intelligence test (known at that time as Binet test and today as IQ test) (1905) was asked for a definition of intelligence, he answered: "Intelligence, it is what my test measures."; if he had been asked for possible grounds of a talent in mathematics, he would have probably answered with the same irony: "Intelligence!". Such an answer would have left teachers skeptical as to the way of dealing with differences in creativity (i.e. the ability to create "something new") of their students when teaching. More precise definitions, presented by psychologists – e.g. Julian de Ajuriaguerra, who introduced the term "over-gifted" – will all be enigmatic to the question of the origin of giftedness; they will stay faithful to its evangelic origin (Matthew, XXV, 14). We are neither psychologists nor neuropsychiatrists; as didacticians we will approach the problem of highly able students independently of any ideology (e.g. of the type “for” or “against”); indeed, even if a didactician is not able to state whether specific educational policy of differentiation is profitable for one or another category of students, his/her work can still contribute to clarification of the intentions and probable outcomes brought about by this organization.

ORGANIZATION OF THE EXPERIMENT

This research uses a larger set of observations whose aim is to study the impact of teachers’ didactical variability (i.e. their ability to organize situations – and therefore knowledge – for teaching arithmetic) on students’ mathematical culture (Novotná, Sarrazy, 2011).

The following paragraphs describe the conditions of these observations.

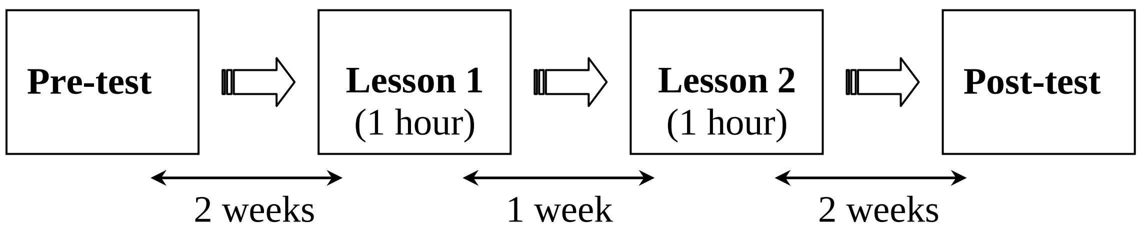
The studied population

The representative sample involved in the experiment consisted of 112 French pupils aged 9-10 from 7 primary school classes. Their school level was evaluated using a standardised test that enabled us to position the pupil’s level in relation to the French school population. The pupils were then divided into three groups corresponding to the criteria developed by the authors of the test: “Good”: the mark (x) in TAS is in the interval $<8; 10>$; “Average”: $x \in <5,5; 8)$; “Weak”: $x \in <0; 5,5)$. There is strong correlation with the teachers’ evaluation (χ^2; $p < .001$).

The conditions of the observation

The conditions of the teachers’ teaching were meant to be as close to their usual work as possible; the conditions were discussed in two perspectives: a) the topic of the lesson; b) the time of the observation. The teachers had to perform two one-hour lessons; there also was a pre-test and a post-test.

The course of the experiment was standard; it proceeded according to the following scheme:



The topic of the lesson

The topic taught was meant to be appropriate to the pupils’ level and simultaneously was meant to be of novel nature in order to avoid deviations possibly caused by what the teachers had already taught, in other words in order to limit the effects of didactical memory of the class (Brousseau, 1997).

The topic selected for the two lessons corresponds to the fourth additive structure of Vergnaud (1983) because it allows integration of the two above described methodological restrictions. This structure works only with positive or negative transformations (“gain” or “lose”) without any indication of the initial numerical status. This is an example of such a problem:

> Lou plays two rounds of marbles. She plays the first round and then the second. In the second round, she loses 4 marbles. After the two rounds she wins 6 marbles. What happened in the first round?

The pre-test and post-test contained 22 problems of this type (selected from 24 types of problems for this type of structure); each problem contained at most two numbers smaller than 10. The level of difficulty of these problems depends on the position of the unknown (possibilities: 1st transformation, 2nd transformation or compound transformation) and on the transformations that can be either of the same sign or of the opposite sign. For example, the above given problem *Lou* is very difficult for 9-10-year old pupils, while the below given problem *Dominika* is much easier, although a non-negligible proportion of the pupils produces an incorrect answer anyway (“*She has* 2 marbles altogether” instead of “*She won* 2 marbles altogether.”)

> Dominika plays two rounds of marbles. She plays the first round and then the second one. In the first round, she wins 6 marbles. In the second round, she loses 4 marbles. After the two rounds she won 6 marbles. What happened in the whole game?

To avoid any influence on the teachers’ organization and structuring of the lessons, the teachers had no access to the evaluation before the post-test – this condition was negotiated as part of the research contract.

This paper does not give us enough space for description of the observed lessons (and it is not necessary for our purposes)[1]: for our purposes we focus on the consequences of pupils’ learning as a function of the initial level (shown in the pre-test). Thus we want to find the possibilities the teachers have (whatever their teaching style is) for differentiation of teaching with respect to their pupils; cognitive abilities.

ANALYSES AND RESULTS

Figure 1 represents the distribution of success in the pre-test:

[1] For a more detailed description of teaching activities focusing on this type of knowledge see (Chopin, 2011).

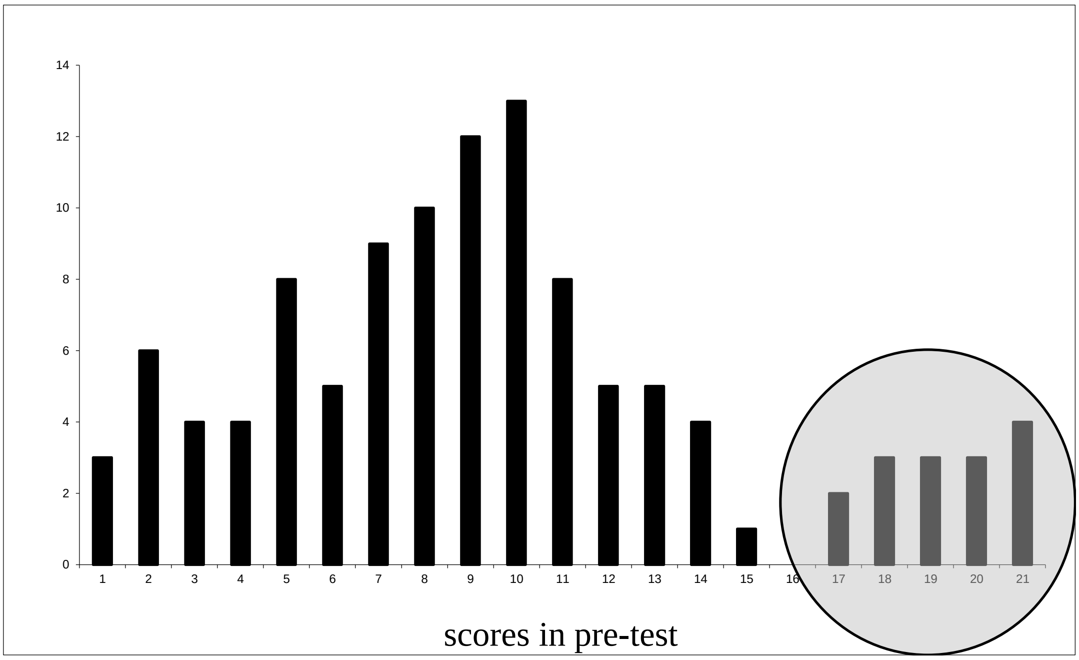


Figure 1: Success in the pre-test

In the following analyses 15 pupils who were successful in the pre-test at least in 17 problems out of the given 21 are considered as highly able.

Who are they?

They represent 13.4% (n = 15) of the participating pupils. There were more boys (73.3%) than girls (26.7%) – out of 51% boys and 49% girls in the whole set of pupils (χ^2= 2.64; ns; p = .11). Their majority belong to socially developed classes (54%), 46% to average classes. All their parents passed secondary school-leaving examinations and 2/3 of them have a university degree. A questionnaire for the families confirmed that the parents' practices in the process of their children's upbringing were flexible (their children could negotiate the rules of life) and "curiosity" and "critical approach" were dominant values in their educational approaches. To put it briefly, these are all students who succumb to rules which, whatever their degree of precision, must nonetheless be applied differently according to the context and situations.

In the context of the class the psychosocial status of these pupils is high in case of 87% of them and their majority are well aware of it (they neither underestimate nor overestimate their abilities). Unlike the majority of pupils, they always ask the teacher as soon as they do not understand something in mathematics lessons. They do not interact more than other pupils (t = 0.11; p = .91) but their interactive profile is significantly different from the rest (χ^2 = 10.74; p = .005): they do not ask to speak, they simply speak.

Theoretical model of the study

Didactical treatment of heterogeneities of competences by the teachers

It is a generally accepted fact that all teaching and learning, at least in the school environment, tries to develop the knowledge of the largest possible number of pupils in the limited amount of time. This development surfaces as a decrease in the number pupils' errors; in other words, as a reduction of heterogeneity of pupils' decisions on how to proceed, answers etc. that are acceptable for the teacher. But the teacher has no tools for *direct* handling of heterogeneities of pupils' differences in their giftedness, the differences in their attitudes to mathematics, of the time and attention they are ready to devote to it etc. What he/she does in his/her teaching in the beginning is that he/she complies with this original heterogeneity, trying to optimise it. In fact, no matter what the level of the class is (weak, advanced or very advanced), a too ambitious a lesson would be too difficult for a considerable proportion of pupils and a too simple lesson would also be unacceptable (loss of time). Individual pupils are often more than happy and comfortable in their positions of "good pupils", "weak pupils" etc. For several reasons, these categories of classification must be considered as functioning of didactical systems as such, independently on the initial abilities of individuals (Brousseau, 1997).

Let us now explore the following two questions:

1. Did the teaching enable learning for the largest possible number of pupils?
2. Is this learning equally distributed with respect to the pupils on different levels?

Results

1. Not all the pupils benefitted equally from the teaching: pupils of the above average and average levels profited most (about 58% of pupils) – see Table 1:

	School level			
	Highly able	***Good***	***Average***	*Weak*
Number	15	**17**	**48**	32
m	0,7	**4.45**	**6.55**	2.32

Table 1: Means of success in the post-test

2. Those good and average pupils who were the worst in the pre-test made the biggest progress in the post-test (and vice versa). This shows a strong correlation between the level of success in the pre-test and in the post-test. This does not hold for weak pupils.

	School level			
	Highly able	***Good***	***Average***	*Weak*
ρ	-.57	- 0,78	- 0,73	-0,34
p.	.02	2.10^{-5}	.0001	.15

Table 2: Correlation of success pre-test/improvement (Spearman's rho)

Conclusion: Teaching is efficient only when we start from the threshold of the initial abilities; this can be called according to A. Marchive (1997) and referring to Vygotsky a "zone of proximal teaching" in which the teacher can teach with reasonable outcomes.

The presented results allow us to acknowledge the following two statements. No matter what the level of the considered class (F(pre-test) = 2.60; p. = .02) is:

- if the difficulty is lower, more pupils advance and heterogeneity decreases (r = -.87; s.; p. <.001);
- if the difficulty is high, the progress pupils make results in an increase of heterogeneity (R = +.83; S.; P. <.001).

DISCUSSION

Would the principle of didactical differentiation (proposing of different things according to the students' initial level) allow us to support the conditions of creativity for all students, i.e. to support their learning?

As indicated in the previous text, regardless of the initial level of the class, any progress in knowledge in the class leads to an increase in heterogeneity. The lower the progress is, the more the heterogeneity is reduced. This implies that grouping pupils according to their level of giftedness does not bring optimisation of their learning, whatever their initial abilities are (there are researches that support this hypothesis – e.g. Duru-Bellat, 1996 ; Mingat, Duru-Bellat, 1997). The fact is that when teaching, the teacher must inevitably differentiate among pupils. We must realize that the fall of very good pupils to weak positions in case they are in large, above average classes will result in loss of courage, decrease in self-confidence etc. This is the price we must be ready to pay for an increase of the average level of the class. The following scheme (Fig. 2) illustrates this phenomenon:

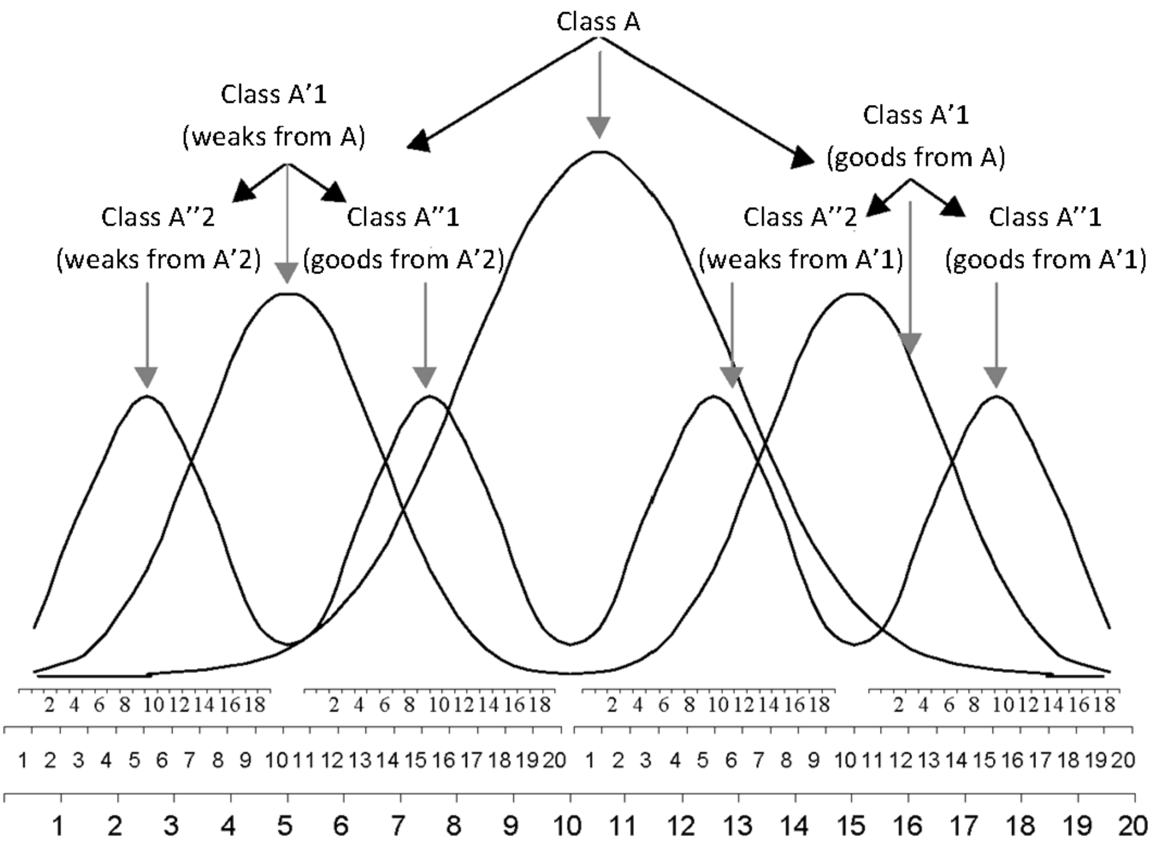


Figure 2: Division into classes according to students' giftedness

The decision to teach in such a way as to improve the performance of the best pupils at the expense of weaker pupils is political. It is not the task of mathematics educators to judge this choice. Their role is to help clarify this situation. As Mc Dermott & Varenne (1995, 343) claim the place that is reserved for pupils from cultural minorities says much more about how our institutions work, about the values they bring than about the expected cognitive priorities. These ideas are "clearly adapted to the functioning and institutions that, across a formal educational system, serve to political and economic goals". This is certainly not the question of glorification of egalitarianism or of radical elitism. We only warn of the dangers of the situation when modern democracies produce categories of individuals who are not able to communicate outside the boundaries of their own cultural community. Our results suggest that research motivated by legitimate concerns about effectiveness and equality is of great potential, as long as it pays sufficient attention to the conditions of organization of such teaching/learning situations in which everybody can adjust the knowledge to make it useful for his/her life. We are fully convinced that there exists a happy medium between acceptation of *indifference to differences* (P. Bourdieu) and its total refusal. It is the task of didactics to show this happy medium or at least to help to clarify it.

The set of these results evokes the fundamental question of orientation of education: Should it orient towards good mastery of algorithms or allow students to be creative in application of these algorithms in new situations (for this is the core of creation: it does not lie in rediscovery of some algorithm but in how the student really applies it in new situations)? It seems that both these orientations must be present simultaneously, which causes a paradoxical relationship: The more the teacher makes his/her teaching algorithmic, the more he/she limits the opportunities for creation for his/her students; the less he/she makes it, the less the students' knowledge is important and the less the students have the opportunity to create something new (as shown e.g. in (Sarrazy & Novotná, submitted to ZDM for 2013) the best students allow themselves to create new relations).

The Theory of didactical situation is born from the theorization and the scientific study of conditions enabling to overcome this paradox; although its recognition in the scientific community is high[i], its dissemination and its use in teacher training remain strongly limited, as it is shown by Marchive (2008). Should it be regretted? Certainly, as teacher training shows to be an important lever allowing teachers to leave this pointless debate. For the teacher, it is fundamental to trust students' creativity, but this pedagogical belief often leaves them helpless when they are to prepare conditions for: pedagogical intention itself is powerless face to face students' incomprehension.

We believe that it is desirable to increase teachers' didactical culture; in fact, if a didactician contributes to clarification of the conditions under which a student may be given the chance to create new knowledge (this contribution does not depend on the student but on the mathematical culture itself), the teacher's responsibility remains to manage the socio-affective conditions that allow the student to get involved in the adventure, which nobody can experience instead of them: the adventure of grasping the whole world in one day on their own, gaining the profit from it. How can one imagine that students would be able to produce something new, had they never had any opportunity to experience it? This is our noble mission: to organize the conditions for such mathematical creativity; the fact that some of them succeed in this adventure better than others is not, as we tried to show, the teachers' responsibility as long as he/she creates the conditions allowing the possibility of this adventure for all students.

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[i] Its founder, Guy Brousseau is the first laureate of the Felix Klein medal for long-life research in the field of mathematics education, awarded by the International Commission on Mathematical Instruction.