SEEING – ACTING – SPEAKING IN GEOMETRY: A CASE STUDY

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The purpose of this paper is to describe and analyse the confrontation and changing processes of frequentation modes (seeing – acting – speaking) of 1st grade high school students (10-11 years old) during two geometric construction tasks. This work was based on a logic analysis of the mathematical concepts involved: the midpoint of a line segment and the circle.

INTRODUCTION

In their research work, Bulf, Mathé and Mithalal (2011) have described how the students' "relationship to knowledge" changes in a learning situation in geometry. For this purpose, they introduce the notion of *frequentation mode* which aims at accounting for the consistency of physical and language dimensions of geometry activity. They insist on the fact that, from the one hand, language practices are not different from other practices, they have no hierarchy (in order to avoid language issues being pushed into the background of the action, especially in geometry) and, on the other hand, language practices are constituent parts of subjects' knowledge rather than mere reflections of pre-existent knowledge. In this context, learning is described as a changing process of frequentation modes to a relationship in line with knowledge objects. This approach seems to echo the hypothesis of a "discursive community" (Bernié, 2002; Jaubert, Rebière, and Bernié, 2003), a French approach analogous to the discursive or communicational one (Kieran, Forman, and Sfard, 2001). In a Vytgoskian perspective, these authors introduce the notion of a school disciplinary discursive communities: knowledge is tightly linked with the community which created it, learning in a school subject area is learning how to act-think-speak somewhat like experts. This means learning how to take position in a social universe characterized by interesting issues, specific objects, material and practices, in particular language practices, learning *Discourse practices* (Moschkovich, 2007). In line with Bulf et al. (2011), we consider it is important to account for three dimensions at the same time – seeing, acting, speaking – to describe in a consistent way how students interact with geometry knowledge objects. The word *seeing*, that we prefer here to the word *thinking* even if they have analogous meanings, underlines the importance of visualization in geometry (Duval & Godin, 2006). As to the word *acting*, we have decided to focus specifically on the artefacts as a didactical variable which is determining in construction situations (Perrin-Glorian, Mathé, and Leclerc, 2013). At last, as to the word *speaking,* we shall intend to describe the specific *language games* (Wittgenstein, 1953) implemented by students and their changing process (Barrier, 2011; Mathé, 2012). However, we do not have the feeling to follow a radical *communicational* way of thinking (Sfard, 2001). Indeed, we are as well interested in the dialogical and social aspects of discourse as in the semantic

dimension of language, *i.e.* its ability to refer to external mathematical objects. The purpose of this paper is to show, from a case study, how a logic analysis of concepts can help to describe frequentation modes and their dynamics. We shall use the language of the first-order logic, i.e. the fundamental logic categories will be those of object, predicate and relation. The extracts analyzed in the following pages are taken from data collected for another work dedicated to issues relating to the use of history of mathematics in classroom. Detailed information on the context of data collection and on students' tasks is available in Barrier, de Vittori, and Mathé (2012). In this study, we selected some sequences of a lesson during which 1st grade students (10-11 years old) in a French secondary school had to draw a square on the playground using unusual artefacts: a chalk and a rope. The construction program given to the students and the expected figure are available in annex. In this problem, students are asked to construct a midpoint (part 1) and a circle (part 2). The lesson lasts just under an hour and takes place outdoors, in the playground. Students are divided in groups of three or four and the teacher moves from one group to the other. The sequences reported have been filmed.

Starting from the logic analysis of midpoint and circle concepts, we shall attempt to bring out *a priori* the potential of frequentation modes which could be considered for the geometrical objects involved in the problem set. This will provide an analysis framework which will be used, during the progress of the sequence, to identify coexisting divergent interpretations of the situation and will help to better understand how they evolve. Our observables will be the students' gestures and procedures as well as their dialogues between them and with the teacher. This work is part of a larger project aiming at creating tools that could account for the geometrical practices both in their physical and language dimensions.

CONSTRUCTING THE MIDPOINT

A priori analysis

The figure of a line segment may be seen in different ways: it may be seen as a part of a straight line or as a couple of points. In the first case, the midpoint of a line segment is characterized by a binary relation between two objects, a geometrical object of dimension 1 – the line segment – and an object of dimension 0 – the midpoint (example of statement of the relation: the midpoint is the point of the line segment that splits it into two parts of equal lengths). In the second case, it can be characterized by a ternary relation between three objects of dimension 0: the end points of the line segment and the midpoint (example of statement of the relation: the midpoint is the point aligned and equidistant with the two end points). These two ways of seeing call for objects which differ in number and nature (in dimension). From a physical action standpoint, there are many possible construction procedures. In the first grades of secondary school, the more usual method consists in using a graduated ruler (to measure the length of the line segment and to plot half of the length from one of the ends). The property explicitly studied in this procedure is the

fact that the middle M of a line segment $[AB]$ checks the equality $AM = ½\ AB$. Questions such as the alignment of the midpoint with the end points or the middle belonging to the line segment are evened out by using the graduated ruler. Another method will be used later: plotting the perpendicular bisector using a ruler and a compass. In the present situation, the rope can be used as an artefact to split the length and check the alignment. The construction requires to explicitly account for both alignment and equidistance properties. It should be noted that the construction program proposed (annex) does not explicitly require plotting the line segment involved. There are several procedures available. We shall describe three of them:

P1. The first procedure consists of plotting the line segment on the ground by stretching out the rope, identifying the ends if necessary and then in folding the rope in two equal parts. The midpoint is obtained by plotting the length from one of the ends. In this case, the property under which the midpoint belongs to the line segment is no more explicit than in the ruler procedure, since the question of alignment (or of belonging to the line segment) is evened out by the plotting of the line segment that is made independently of the construction of the midpoint.

P2. If the line segment has not been previously plotted, the rope can be laid in a straight line on the ground and then be folded in two parts by moving only one of its ends, the other end staying in the very same place. Therefore, the midpoint is placed at the end of the new line segment thus obtained. Theoretically, this procedure does not call for the alignment or line segment belonging issues, but the question arises from a practical standpoint, since it is difficult to move half of the rope while the other end stays still.

P3. The third procedure explicitly accounts for the alignment property. It starts by laying the rope on the ground as described in the procedure P2 and by identifying the ends. Then the rope is folded in two equal parts and the new length is used to plot an arc of a circle the centre of which is one of the ends of the line segment. The midpoint is obtained by determining, using the rope, the point on this arc of circle which is aligned with the identified ends (the line segment can be plotted or not).

We consider that the distinction made between the different conceptions of the notion of midpoint, in terms of binary or ternary relation, in addition to the *a priori* analysis of possible procedures, may contribute to demonstrate the potential of frequentation modes which could be considered for the midpoint (of a line segment) object. In fact, we assume that during the solving problem process some practices will develop (ways of acting and speaking), not necessarily homogeneous, but consistent for a given student at a given time. The following *a posteriori* analysis should identify the frequentation modes in which the students stand, detect the possible coexistence of different frequentation modes and intend to better understand how the change towards a frequentation mode of the midpoint notion in line with the school expectations at this educational level operates. It should be noted that the main difference between the possible interpretations of the problem mainly lies in the way

of seeing the "line segment" figure (nature and dimension of geometrical objects handled). However, the midpoint construction procedures using a rope will not necessarily differ, whether the line segment is seen as a part of a line or as a couple of points. Therefore, it seems necessary, for our work's sake, to simultaneously account for the gestures and the physical actions of students as well as the students' discourse between them and with the teacher about these actions.

A posteriori analysis

In this paper, we shall focus on the physical and dialogical practices of a group of three students (E1, E2 and E3) and their interactions with the teacher (H) about the construction of the midpoint of line segment [*OE*]. The three points *O*, *E* and *I* are identified by a cross and by their relevant letter on the playground. The line segment [*OE*] is not plotted and the three points *O*, *E* and *I* do not seem to be aligned. Then, the teacher intervenes and asks the students to explain how they have proceeded. E1 "shows" the way they used to construct the point. He starts by joining the two ends of the rope, joining both his hands to openly show the half-length obtained. This means that the group perceives the length constraints imposed on the construction of the midpoint. Then he lays a part of the rope on the ground. An end of the rope is placed in *O*, while the other end stays in E1's hand and the rope is laid so that it passes by *O* and by *I* (but it does not pass by *E*). Now, he folds the end he holds towards point *O*, without exerting any other pressure on the rope. It seems that this group has used procedure P2, in a more or less successful manner. The interactions proceed in the following way:

H: No, no, but ... have we got a means to check if it is the midpoint?

E1: Why, yes

H: What could we do?

E1: Plot a line.

H: A line?

E2: [Inaudible]

H: Check there... How, how did you place your rope to check that this is the midpoint? [Pause] How will you proceed?

E3: We lay it/ [E3 points his finger towards *O*]

E1: We lay it there/

H: Therefore, we put an end here and then the other end/ [E1 puts an end of the rope in *O*]

E3: We fold it

H: Yes, and the other end? You must stand...

E1: We fold it like that [E1 follows the procedure described above]

H: Well, this is not what I want

We can assume that H checks the control procedure which consists of using the rope as an artefact to check that the three points *E*, *O* and *I* are aligned. As for the students, they seem to be in a frequentation mode that is definitely different from the notion of midpoint. They focus on the distance constraints and they only consider global perceptive retroactions which cannot invalidate their construction strategy. This misunderstanding appears in the form of language interactions. For example, when H says "and then *the other end*/" then goes on with "Yes, and *the other end* ", it seems that he expects an answer with something like an adverb of place to specify where the other end of the rope should be placed. The students' answers are in the action field, to fold the rope in a certain manner, rather than in the place field. This extract shows how language interactions may be a place of confrontation between conflicting frequentation modes and a (attempt of) negotiation towards a shared frequentation mode. For example, when the teacher repeats the E1's statement "we lay it there" in "Therefore, we put an end here and then the other end", he attempts to direct the students' look towards the ends of the line segment and introduce the end *E* as a reference of an adverb of place. Thus, he intends to lead students towards an interpretation of the "midpoint" object defined by a ternary relation between three points. The technique implemented by the teacher aims at pointing out that point *E* should be taken into consideration. The purpose is to set a shared objects field from which construction language games could be compared. Nevertheless, students are not able to lay on language indicators used by the teacher and they do not recognize the specific form of language game he wants them to play. Finally, this misunderstanding leads the teacher to artificially put aside the strategy of this group. Therefore he decides to introduce by himself the third point necessary to make them shift to the punctual standpoint:

H: [H puts his forefinger on point E] Yes but here, in relation to this point, is there a means to check that your point placed there will be the middle point, the midpoint of your line segment?

E3: Well, we plot, er...

H: We plot?

E3: Well, we plot er... the rope.

H: Yes we shall plot (…)

However, it is difficult for students to use the rope as a geometrical artefact to plot straight lines. So far, the rope has been laid on the ground in an approximate straight line, without exerting special pressure on its ends. Now, the teacher takes over a more important part of the problem. He uses the language to simultaneously set in action the three points the alignment of which is to be questioned and clarifies the fact that they must be linked by a specific relation:

H: If your point is the midpoint, how should these three points be?

Es : Er...

E1: On the same straight line

H: On the same straight line, well then have we got a means to check your three points are really on the same line? What can we do?

E1: Oh no, there are like this! [E1 shows that the points are not aligned]

H: Well, how can we check then, how can we be sure it will be placed correctly? [pause] […]

H: [...] You cannot see how we can check the points alignment?

E3: Er… no

H: Well, your task will be... You have to find the way, just think, sort it out yourselves, find how to check that your three points are correctly aligned, that's all [H goes to another group].

This time the students see the necessity to align the points, thus focusing on the line segment, to the exclusive consideration of the lines and lengths by a punctual look, inducing a possible questioning on the points alignment.

In this first example, we have tried to point out the consistency between the modalities of physical action, the discourse and the way of looking at the figure for a given group of students, even when this consistency is disturbed by the teacher's intervention. Let us now present a second example.

PLOTTING A CIRCLE

A priori analysis

From a logical and mathematical standpoint, the circle can mainly be seen (of course, there are many other characterizations of the circle, cf. Artigue & Robinet, 1982) as:

- a set of points characterized by a relation: the fact they are at a given distance (radius) from a given point (centre). This representation corresponds to a plot using a compass or a rope, but also a "point-by-point" plotting (plotting multiple points at a given distance from the centre, then plotting a line if necessary, or linked line segments)

- a continuous line with constant curving. This vision of a circle is hardly operational except for freehand plotting (it can be combined with plotting a few points or few diameters then applying the point-by-point plotting described above), this characterization can also be used for checking a freehand plotting (or using artefacts if necessary)

- the given length line which contains the largest surface area (not quite operational, somewhat corresponding to the circle of a children's dance)

- a line with infinite number of axes of symmetry (not quite operational but it can be used to check during plotting)

It should be noted that the first characterization calls for a relation between objects of dimension 0 (points, including the midpoint which is "exterior" to the graph) whereas the three other points use properties applicable to a single object of dimension 1 (the line).

Of course, the rope can be used to plot circles (or arcs of a circle). This construction requires to follow the same preliminary steps as for plotting with a compass (this artefact is almost always used for plotting circles in a classroom): decision of the radius length to be used (if necessary, selection of the line segment to be plotted, or modalities of length measurement if the length is given with a numerical value) and of the centre around which the circle must be plotted. If the rope is used to plot, it might be difficult to hold one of its ends in a fixed point during rotation. With a compass, when the space between legs has been fixed, the equidistance property of the points of the circle, or from the line to the centre, is accounted for by the stiffness of the artefact itself. In the context of plotting with the rope, this property is tightly linked with the fact that the rope must be held tight during the whole plotting process. It is physically felt by the student who makes the plot circle and it is the required condition to plot the circle. Plotting with a rope usually corresponds to plotting in the meso-space, whereas the compass is commonly used in the meso-space of the sheet of paper. This parameter induces different gestures: plotting with a compass requires hand work, while plotting with a rope requires moving the body and the arms and sometimes needs the intervention of two students (one student keeps one end of the rope on the centre of the circle whereas the other holds the tight rope and draws the required circle with the other end).

In this case, the links between characterizations and modalities of construction clearly appear, as well as the links between the characterization and the nature of the circle object. The objects explicitly or implicitly handled and the nature of their relations (binary relation, property, etc.) differs from one characterization to the other. What is considered: the centre? the radius? a diameter? Is the circle seen as a set of points? A line?

A posteriori analysis

In this part, we shall focus on the analysis of the sequence with the work of a second group, again made up of three students (C1, C2 and C3). This extract begins when the two first steps of the construction program have been completed (plotting points O, E, and the midpoint I of the line segment [OE]). The students have to plot the circle with the line segment [OE] as a diameter.

1. C: how do you want us to plot the circle?
2. C: er, you draw a round shape like (...) [inaudible]
3. C: er, you draw a normal round shape since after, we have no compass, therefore...
4. C: freehand?
5. C: well yes, I think it is like that.

6.	C2:	may I do it?
7.	C1:	No, please wait, wait [C1 joins the four ends of the line segments with his hand.
8.	H:	[H comes to the group] Well ! Well ! What are you doing to me, guys?
9.	C1:	This is not a circle
10.	H:	What are you doing?
11.	C2:	We are supposed to plot a circle
12.	H:	You are plotting a circle?
13.	C3:	Yes
14.	H:	Hum… [sceptical]
15.	C1:	This is not a circle
16.	H:	A circle, what is it? What is a circle?
17.	C3:	Er…
18.	C1:	Er, it is a...
19.	C2:	A circle
20.	C1:	[Laugh] It is a circle, er... There is a diameter and er…
21.	C2:	And a radius [inaudible]
22.	H:	Try to come back to the origin. If I tell you what is the circle with a centre O and a 3 cm radius?
23.	C1:	Well, it's a circle
24.	H:	The circle has a centre O and a 3 cm radius. [pause]
25.	H:	The definition, you've got it in the lesson you should have learnt. If you didn't, you can see how you get stuck now… therefore, is formed by what? [pause]

Without their usual artefacts the students cannot instinctively adopt a mathematical frequentation mode. They suggest to draw "a round shape" and even a "normal round shape", i.e. a round shape which does not refer to mathematics but rather to that they usually use outside the specific approach of spatial issues raised in geometry. This justifies freehand plotting here. The standpoint on the circle adopted here is that of a rounded shape made of a line (a closed line, characterized by its constant curving and/or its symmetries for example). This is a global rather than a local point of view, since it calls for lines and not for points (and the centre of the circle is not evoked). The teacher's interventions can be seen as attempts to (re)-position the students in a frequentation mode of the circle object which is more in line with the school mathematical expectations, to guide them to express their practical concerns through a mathematical questioning on the properties of objects involved and artefacts, using a language suitable for the school mathematical context. This background movement the teacher attempts to bring about is stimulated by the questions: "A circle, what is it? What is a circle? ". Raised by the teacher, these questions call for a change in the students' way of seeing. Moreover, the teacher said before "What are you doing to me" (line 8) and not "What are you doing". The use of pronoun "me" can be interpreted as a sign of a close relationship or complicity but also as a way to stress

the didactical dimension of practices involved 1. The same phenomenon is repeated later with the question "If I tell you" (line 22); H speaks as a mathematics teacher, and expects an answer in the school context. Likewise, the questions on the nature of the circle are not only or mainly aimed at obtaining a definition of the circle in return. The objective is to lead the students to "see" the circle as it is usually seen in the school mathematical context. Some of the answers given by C1 and C2 may seem to be tautological (C2: "A circle", C1: " It's a circle", C1: "Well, it's a circle") and useless in the context of knowledge, students seem to be aware of it, but it is not the case if they are analyzed considering how the dialogue works and how the practices are inserted in the required context. These language interactions show a change in position. This movement is also revealed by the fact that, in other following answers, terms which are specific to mathematical vocabulary are introduced (centre, diameter and radius). Another indication of a change in the frequentation mode appears in lines 22 to 25. With the term "the-circle-with-a-centre-X-and-a-radius-Y" (used twice in an identical manner) and the explicit allusion to the « definition » of the circle, the teacher clearly introduces a formal dimension (Hache, 2012), above all in relation to the situation of the exercise and the supposed frequentation mode of students. Besides, from the knowledge standpoint, the teacher, by using the words "centres" and "radius" for example, refers to the prevailing definition of the circle in the school context, i.e. the circle seen as a set of points placed at a same distance of a given point. As already mentioned, this characterization calls for a property which is made natural by using a compass in the usual situations of plotting. On the other hand, it differs from the instinctive characterization adopted so far by the students, which rather seemed to lay on the circle as a line, characterized by its constant curving. In practice, shifting from one conception to another is not instinctive (Artigue & Robinet, p. 49), all the more as the characterizations appeal to students to have different looks on the figures (Duval & Godin, 2006). The last teacher's intervention in the above extract can be understood as an inducement to fit the way of seeing ("therefore, is formed by what?") with the way of speaking ("the circle with a centre O and a 3 cm radius").

CONCLUSION

The purpose of this paper was to describe and analyse the confrontation and changing processes of frequentation modes of 1st grade students (10-11 years old) during two geometric construction tasks. This work was based on a logic analysis of the mathematical concepts involved: the midpoint of a line segment and the circle. In both cases, according to the adopted standpoint, the mathematical concepts can be described from different categories of logic (property, binary or ternary relation) on objects different in number and nature. This analysis, although it was quite brief, seemed to be useful to consider the consistency and practical harmonization of the three dimensions "seeing – acting – speaking" we called for to describe the frequentation modes (Moschkovich (2007) would maybe have said a *Discourse*). We could thus observe that the change in the way of seeing ("Oh no, there are like this!")

of the students who worked on the construction of the midpoint was produced by the teacher's language action aimed at setting the objects *ends of the line segment* as references for some words in the language games specific to the geometry practice at school. This change in their way of seeing is associated itself with new possible uses of the artefact *rope* (physical dimension of geometry practice). As for the plotting of the circle, we attempted to show how the language practices could be linked with the plotting methods. It seems that the expression "a normal round shape" can be related to extra-school practices which justify the "freehand" plotting that we compared with a global vision of the circle as a rounded shape. If this approach is somewhat justified, the related vision is not that in use in mathematics at secondary school. On the contrary, the language game which calls for the expression "the circle with a centre O and a 3cm radius", and which is initiated by the teacher, introduces some elements required for invoking a punctual standpoint on the circle, in particular the centre of the circle. This centre, exterior to the line actually plotted, must be taken into consideration to implement the techniques which call for the equidistance relation.

We are just at the start of our research and we are not sure to be able to offer a pertinent discussion of these results. Nevertheless, we will try to situate it inside the today well-established *discursive framework* in mathematics education (Sfard, 2012). Our feeling is that the former semantic and dialogic perspective could be one way to consider both social and external aspects of language. Mathematical language games could be *outdoor games* (Hintikka, 1996), i.e. games involving the objects of the language one speaks. Analysis of communication quite often emphasizes interpersonal interactions. In this work, we think it necessary to integrate a specific focus on the interaction between students and external (even if dialogically constructed) mathematical objects, with the help of a logical analysis of the concepts at stake. Of course, all of this is nothing new. For instance, the Theory of Didactical Situation tradition in France has a long time ago pointed the educational interest of the students-*milieu* interactions (Brousseau, 1997) and inside the communicational approach, Sfard (2001) clearly accounts for the "object-level aspects of discourse". We only hope that this research, relying on logical analysis, could contribute to the content related dimension of language games understanding.

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ANNEX

Construction program and example of the expected figure

Stretch out a rope the length of which must correspond to the side of the square to be constructed.

On the ground, mark its ends O and E and its midpoint I.

Plot the circle with a diameter [OE] and circles with radii [OE].

These two large circles are crossed in U and V.

Stretch out a rope between U and V.

Mark as N and S its intersection points with the small circle.

Points U, N, I, S and V are aligned in this order.

Plot the circles with respective centres E, O, N and S the radius of which should measure half the EO length.

These four large circles are crossed two by two in A, B, C and D.

These four points are the vertices of the square.

Follow these instructions and construct a square using the artefacts given to you.